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Article

The Lorentz Force and the Nature of Charge from a Photonic Toroidal Vortex Model

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Abstract

In 1865, Maxwell characterized a magnetic field line as an axis of angular momentum. It is shown why this cannot provide an *explanation* for the migration of charge and the generation of an emf in a conductor moving perpendicularly through these lines. The Lorentz force deflection is instead derived from the photonic toroidal vortex (PTV) model developed by Clarke (2025). The model defines a mass in terms of optical orbital angular momentum (OAM) with a specific beam waist, and here it is shown how motion towards a magnetic momentum field line with a gradient creates a change in momentum perpendicular to it, resulting in an excess or deficit of action in the mass's OAM circuit. This is redistributed into motion along the OAM axis. The mean value of these redistributions, averaged over all OAM positions in the PTV rotation, results in the Lorentz force deflection. In the process, a geometrical nature of electric charge is suggested in terms of a combination of OAM and toroidal rotation senses.

Keywords: string theory; Lorentz force; photonics; optical spin angular momentum; optical orbital angular momentum; photonic toroidal vortex; charge

1. Introduction

Most graduates of physics are familiar with the well-known Lorentz force law

$$\vec{F} = q\vec{v} \times \vec{B} \quad (1)$$

where q is the charge passing through a magnetic field \vec{B} with velocity \vec{v} . However, while it provides a 'rule of association' the deflection effect, it offers no insight into the geometrical nature of charge.¹ It merely describes what is seen on a macroscopic level. It also turns out that there is a crucial inconsistency at the heart of Maxwell's derivation of the Lorentz force (as it was later known) in that a magnetic field line serves as an axis of angular momentum thus

$$\vec{B} = \nabla \times \vec{A} \quad (2)$$

where \vec{A} is the vector potential or the momentum per unit charge. To see this, for a rotation with angular velocity ω around the z axis, since the velocity $\vec{v} = \vec{\omega} \times \vec{r}$ then $\nabla \times \vec{v} = 2\omega\hat{k}$. The momentum per unit charge can be written as $\vec{A} = m\vec{v}/q$ so $\vec{B} = \nabla \times \vec{A}$ becomes $\vec{B} = 2m\omega\hat{k}/q$. For a symmetrical object, the angular momentum $\vec{L} = I\vec{\omega}$ where I is the moment of inertia. Then with $\vec{\omega} = \omega\hat{k}$ we have $\vec{B} = 2m\vec{L}/qI$. That is, the vector \vec{B} shares an axis of rotation with angular momentum \vec{L} .² The objection we shall raise here against the nature of \vec{B} creates a need for a re-

¹ A 'rule of association' merely relates the causal variables to the effect variable with no attendant mechanism. Ampère's law and Coulomb's law are similar rules.

² In §2154 of his *Experimental Researches* Faraday discusses the Faraday Effect in which magnetic field lines passing through a diamagnetic rotate the plane of polarized light passing along these lines. He says "the character of the force thus impressed upon the diamagnetic is that of *rotation* [...]" [1]. However, the magnetic field lines are not necessarily conveying a rotation that they possess. They could be causing the diamagnetic to induce one.

examination of what this vector represents. In this paper, the principles of a recently developed photonic toroidal vortex (PTV) model [2] will be used to obtain Eq. (1), identify the magnetic field line \vec{B} as a flow of field momentum, and reframe the nature of charge in terms of rotation combinations of optical orbital angular momentum (OAM) and toroidal rotation.

Optical SAM was demonstrated by Beth in 1936 who showed that light exerts a torque on a rotating half-wave plate as its circular polarization is reversed [3].³ Subsequent experiments managed to replicate the effect [4–7]. Optical OAM can be created in the laboratory by diverting the trajectory of optical SAM into a pseudo-cylindrical beam with a phase that is dependent on azimuthal angle.⁴ Spiral wave plates [8], holograms [9–11], and cylindrical lenses [12–15] have all been used to create a helical phase structure from the diversion of linear rays. In 1992, Allen et al. predicted that in the paraxial approximation, a beam that possesses both SAM and OAM should convey a torque

$$T = P_{abs}(l + \sigma_z)/\omega \quad (3)$$

where P_{abs} is the power absorbed by a material fragment (proportional to the number of photons absorbed per second), $l\hbar$ is the OAM per photon with l a non-zero integer, $\sigma_z = \pm 1$ for left- and right-circularly polarized light, and ω is the angular velocity [14]. The transfer of OAM from a linearly polarized LG mode laser to trapped CuO particles was subsequently observed by He et al. [11],⁵ and by Friese et al. using a combination of SAM and OAM [16].⁶ The latter work corroborated Eq. (3) in that when SAM is introduced to a rotation frequency of a fragment created by OAM, it increases when the spin and orbital rotations reinforce, and decreases when their rotations are in opposition [16]. A more efficient experiment that allowed the whole of the beam to participate was performed by Simpson et al. who used weakly absorbing Teflon fragments $2\mu\text{m}$ in diameter in a three-dimensional optical spanner [15].⁷ In 80% of their trials, when the SAM and OAM reinforced the fragments speeded up, most of them doubling their angular velocities. The fragments became stationary when the SAM and OAM rotations were in opposition. So both optical SAM and OAM have been demonstrated to produce mechanical effects.

A photonic toroidal vortex (PTV) has been created in the laboratory that allows optical OAM to run around the surface of a toroid and occupy the whole surface [17]. Without recourse to either quantum mechanics or classical electrodynamics, a PTV theory has been developed based on the OAM of a helical string that obtains an accurate fine structure spectrum for the hydrogen atom [2]. This theory defines an internal potential to accommodate the fact that electron vortices have been found to adopt discrete OAM states in the absence of an external potential [18–20]. More recent work has extended it the hyperfine structure [21]. In the present work, we make use of the principles introduced in the internal-potential PTV theory [2] to present a visualizable *explanation* of the Lorentz force deflection and to suggest the geometrical nature of charge. The consequences of this identification for the electrical and magnetic momentum fields in PTV theory [2,21] are discussed.

2. The Inconsistency in Maxwell's Derivation

The first equations for the deflection of a charge passing through magnetic field lines were given by Maxwell in March 1861 ([22], Eq. (77)) in Part II of a four-part paper 'On physical lines of force'.

³ Beth found that with a clockwise rotating plate, the incident rcp wave polarization changes to lcp, the wave frequency is reduced, and the plate gains angular momentum. Also, lcp turns into rcp, the wave frequency is increased, and the plate loses angular momentum.

⁴ The theory developed in Ref. [2] suggests that at an atomic level the SAM trajectory *curves* around a notional tube.

⁵ The absorbing particles were black superconductor ceramic powder of size $1 - 2\mu\text{m}$ soaked in kerosene.

⁶ The particles were CuO of size $1 - 5\mu\text{m}$ soaked in kerosene.

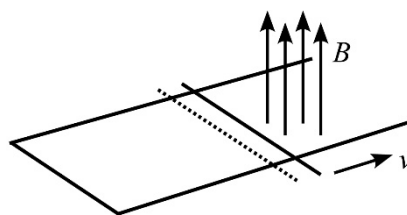
⁷ An optical spanner uses Laguerre-Gaussian laser modes with optical OAM to obtain a tightly focused beam that can trap particles.

Four years later, Maxwell changed the signs on the time derivatives of the magnetic vector potential components F, G, H from positive to negative to obtain Eqs. (4) [23]. Previously, Maxwell had introduced his displacement current which required a sign-change in the vector potential in order to derive the wave equation.⁸ His equations for the components P, Q, R of the electromotive force \vec{E} , he gave as

$$\begin{aligned} P &= \mu\gamma \frac{dy}{dt} - \mu\beta \frac{dz}{dt} - \frac{dF}{dt} - \frac{d\Psi}{dx} \\ Q &= \mu\alpha \frac{dz}{dt} - \mu\gamma \frac{dx}{dt} - \frac{dG}{dt} - \frac{d\Psi}{dy} \\ R &= \mu\beta \frac{dx}{dt} - \mu\alpha \frac{dy}{dt} - \frac{dH}{dt} - \frac{d\Psi}{dz} \end{aligned} \quad (4)$$

where α, β, γ are Maxwell's components of the magnetic intensity \vec{H} , μ is the coefficient of magnetic induction, F, G, H are his components of the magnetic vector potential \vec{A} (magnetic momentum per unit charge), and Ψ is the electric potential ϕ . When *A Treatise on Electricity and Magnetism* appeared in 1873, Maxwell derived Eqs. (4) by making use of Faraday's law of induction in the context of the magnetic effect of a secondary circuit on a primary ([25], Art. 598), see Yaghjian (2020) for an exposition in modern notation [26].⁹

In his analysis, Maxwell considers a straight wire sliding perpendicularly on two parallel rails that are connected at one end to form a closed circuit, see Figure 1. If the circuit is considered to be in a specific plane, the magnetic field lines \vec{B} run normal to that plane. The local length of the wire, velocity v of the wire, and the magnetic field lines are mutually perpendicular. The aim is to evaluate the sum of the scalar products of the time-rate-of-change of the magnetic vector potential \vec{A} — which Maxwell calls “electrokinetic momentum” ([25], Art. 590) — and the circuit length increment in a line integral. It is essentially a sum of increments of the time-rate-of-change of action per unit charge taken around the circuit and this is Maxwell's starting point. The electromotive force is the rate of decrease of action around the circuit ([25], Art. 579). To arrive at the Lorentz force law (as it was later called), Maxwell's method relies on the representation of a magnetic field line as the axis of rotation of an angular momentum per unit charge using $\vec{B} = \nabla \times \vec{A}$ because without this relation he cannot introduce the vector \vec{B} into the law [26]. However, we shall now see that although in practice there is an emf generated by a change in field lines, in theory, a rotation of momentum per unit charge around these field lines cannot account for it.¹⁰ This suggests that these rotations do not actually exist.



⁸ Maxwell's displacement current appeared in Part III of his work [24]. It was meant to represent a displacement of charge in an aether that was later shown to be superfluous. It's only use is to derive the wave equation.

⁹ In 1889, Heaviside quotes “Maxwell's ‘electromagnetic force’” as $\mathbf{F} = \mu_0 q \mathbf{V} \mathbf{u} \mathbf{H}_0$, where he uses \mathbf{V} to denote a vector product [27].

¹⁰ It is irrelevant that when we invoke Stokes' law, confine ourselves to the \vec{B} field lines inside the closed-circuit aperture, and suppose there to be rotations around the lines (some parts of which cancel out) that we obtain a reasonable prediction of the emf. Our concern here is with an *explanation*, and these rotations fail to provide one.

Figure 1. Straight wire sliding at speed v on two parallel wires that form a circuit, to pass through perpendicular magnetic field lines \vec{B} .

Consider an increment of conducting wire with a field line adjacent and perpendicular to it on one side, Figure 2a. In order to change \vec{A} , the momentum per unit charge, either a change in field strength is introduced along the direction of \vec{B} , or relative motion through space substitutes the line \vec{B} with another one that takes up the same position relative to the conducting increment. The part of the change in angular momentum rotation adjacent to the wire communicates momentum to the charges generating an emf. Now let us add a parallel \vec{B} field line adjacent to the wire on the opposite side, Figure 2b. The two change-in-momentum rotations now cancel out and there is no emf. Increments of wire can be added to our conductor end to end to form a closed circuit with a similar field line on each side of each increment. Since all increments behave alike, there is no net emf generated.¹¹ Nevertheless, when in practice the field lines are changed around a conducting loop — and they must manifest both inside and outside of the closed circuit — an emf is *measured* to occur. This means that the notion of a vector potential \vec{A} circulating a field line \vec{B} is incapable of accounting for it.¹² So there needs to be a re-examination of the nature of a \vec{B} field line emanating from a magnetic north pole or solenoid.

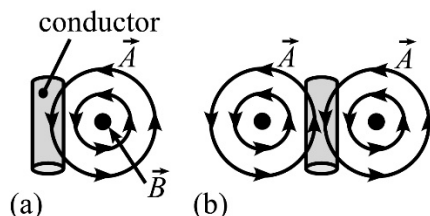


Figure 2. a A changing magnetic field line \vec{B} (running out of the page) with circulating changing vector potential \vec{A} cutting an increment of conducting wire producing an emf. **b** When the \vec{B} field lines are on both sides of the conducting increment there is no net emf due to opposition of the vector potential rotations.

3. Principles of the PTV Method

3.1. Definition of Rest Mass

A model for the structure of a single photon has been proposed in which a string follows a helical trajectory at speed $\sqrt{2}c$ and rake angle $\pi/4$ so that the linear and azimuthal speed are both c , see Figure 3a.¹³ This rotation is denoted as Sp-1 and Figure 3a shows a helical string (HS) that is left-circularly polarized. In this model, a single photon consists of an advancing array of such tubes [29]. The notional tube in Figure 3a is then given a helical trajectory around another notional tube at speed c to produce optical OAM denoted as Sp-2. This has a linear speed βc . The Sp-2 rotation also allows a definition of the electron and proton rest mass which is defined by

¹¹ If the increments form a closed loop and the field lines are equally spaced there should be more curve-contiguous lines outside the curve than inside and the charge migration would be oppositely directed to that observed.

¹² I suspect that this objection had already occurred to Heaviside. He suggested abandoning Maxwell's $\mathbf{E} = -\dot{\mathbf{A}} - \nabla\Psi$ and $\text{curl } \mathbf{A} = \mu\mathbf{H}$ in favour of his own construction $-\text{curl } \mathbf{E} = (4\pi g + \mu d/dt)\mathbf{H}$, where g is "a physically non-existent quantity" and announced that "The Ψ and \mathbf{A} are murdered, so to speak, with a great gain in definiteness and conciseness" [28].

¹³ The original model was an array of non-rotating screw threads advancing at speed c , but Ref. ([2], footnote 8; 23) suggests that an array of strings each following a helical trajectory has greater possibilities. It is the latter model that is adopted here.

$$m_o = \frac{h}{2\pi r_{2o} c} \quad (5)$$

where r_{2o} is the rest mass tube radius with $\beta c = 0$. The electron rest mass occurs for $r_{2o} = 3.861\,592\,68 \times 10^{-13}$ m while the proton rest mass has $r_{2o} = 2.103\,089\,104 \times 10^{-16}$ m ([2], Eq. (1)).

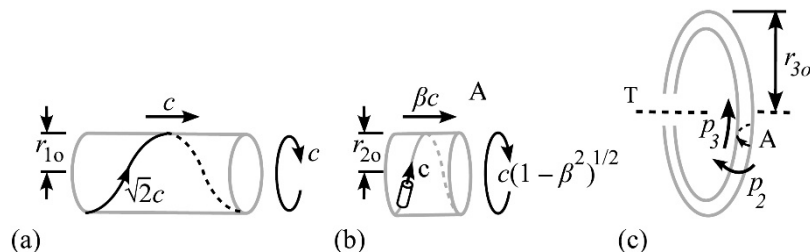


Figure 3. **a** Construction of a circularly polarized ray or optical SAM (denoted as Sp-1) in which a string follows a helical trajectory around a notional tube at speed $\sqrt{2}c$. **b** Construction of optical OAM (denoted as Sp-2) in which the notional tube in Figure 3a follows a helical trajectory at speed c around a larger notional tube, with linear speed βc . **c** A photonic toroidal vortex with the structure A in Figure 3b rotating around a toroid with poloidal (Sp-2) momentum $p_2 = \beta c$ and toroidal (Sp-3) momentum $p_3 = c(1 - \beta^2)^{1/2}$.

3.2. Momentum Fields

The model presented here assumes that both the Sp-2 and Sp-3 rotations give rise to momentum fields in the surrounding space with initial resultant speed $\sqrt{2}c$. There is a radial component to the field with speed c (Figure 4a) that rotates with the Sp-2 momentum field (Figs 4f and 4g). An equal partitioning between the radial and the resultant of the Sp-2 and Sp-3 rotational momenta occurs at the emission point A in (Figure 4b and 4e).¹⁴ The radial and Sp-2 speeds diminish proportionally with distance ([21], Eq. (9)) so that a 45° spiral angle to the line AB joining source and target Sp-2 circuits is maintained during field transit (Figs 4c and 4f). The Sp-2 magnetic momentum field magnitude is given by $p_{2f} = \hbar/r_{2f}$ where r_{2f} is the field radius from the Sp-2 center ([2], Eq. (21)). Since $\hbar = mr_{2o}c(1 - \beta^2)^{1/2}$ ([2], Eq. (6)) – $m = m_2$ is given by Ref. [2, Eq. (10)] – then the Sp-2 field momentum varies (due to speed variation) as

$$p_{2f} = mc(1 - \beta^2)^{1/2} \frac{r_{2o}}{r_{2f}}, \quad m = \frac{m_o}{(1 - \beta^2)^{1/2}} \quad (6)$$

where r_{2o} is the radius of the Sp-2 tube and m is the relativistic mass ([21], Eq. (9)). So if $\beta \ll 1$, given that the azimuthal speed varies inversely as r_{2f} ([2], Sec. 2.7.2), the field emission begins its path at A through space at approximately angle 45° to the Sp-2 rotation-tangent in a plane almost perpendicular to the Sp-2 axis.¹⁵ If we construct a straight line AB between the Sp-2 centers of the source and target (Figure 4f), due to the rotating radial vector in Figures 4e and 4g, the \vec{p}_{2f} momentum vector is maintained at 45° to this line. This supports an earlier assumption about the 45° orientation of the proton magnetic momentum field when it strikes a distant electron Sp-2 circuit, one that is crucial to the hyperfine calculation for the hydrogen atom ([21], Figure 4b).

¹⁴ Since $|p_{3f}| \ll |p_{2f}|$ and $\beta c \ll c$, Fig. 4 is shown neglecting the participation of the Sp-3 speed and momentum in the partitioning.

¹⁵ The Sp-3 drift speed βc is taken to be negligible here.

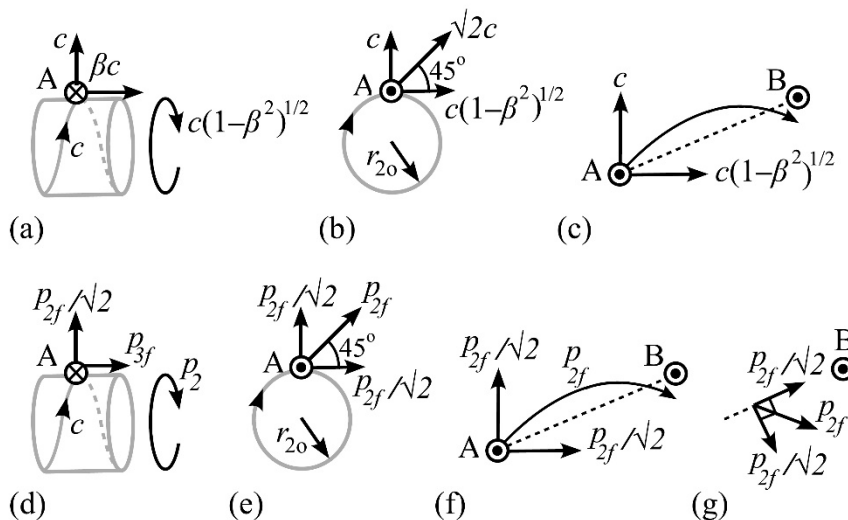


Figure 4. Velocity and momentum field components of the Sp-2 and Sp-3 rotations. **a** The Sp-2 circuit in Figure 3b, when situated in a toroid, emits a field throughout its trajectory, with radial speed c , Sp-3 speed βc , and Sp-2 speed $c(1 - \beta^2)^{1/2}$ into the page (cross). **b** Right-end elevation of the Sp-2 in Figure 4a in which the component at A out of the page (dot) is βc and the resultant projection initially runs at speed $v_f = \sqrt{2}c$. **c** Trajectory of the field emission at A towards a target Sp-2 circuit at B, in which the radial direction rotates. Both the radial and Sp-2/Sp-3 speeds diminish proportionally with distance to produce a momentum p_{2f} approximately 45° to the line AB. **d** Momentum analogue of Figure 4a. **e** Momentum analogue of Figure 4b in which the field momentum is equally partitioned between the radial and the resultant of the Sp-2/Sp-3 directions. **f** Momentum analogue of Figure 4c. **g** Detail of B in Figure 4f in which the radial field component has rotated with the Sp-2 field rotation to point towards B and maintain p_{2f} at 45° to the line AB.

3.3. Interaction Between Mass and Radiation

In the PTV model, an electron is represented as a Sp-3 toroid, the Sp-3 rotation being necessary to provide the charge. Its Sp-2 rotation carries the mass. Consider a Sp-2 circuit moving with momentum \vec{p}_e along its axis. This would manifest as Sp-3 rotation in an electron PTV. If a photon with momentum \vec{p}_p strikes it as the rotational momentum in optical SAM or OAM, the electron momentum along its axis changes to $\vec{p}_e + \vec{p}_p \cdot \hat{p}_e$, where the latter term is a scalar product. The Sp-2 momentum rotation in the mass does not participate in mass–radiation interactions. Only the radiation momentum component along the Sp-2 axis (as Sp-3 rotation in a toroid) is to be considered.

3.4. Interaction Between Mass and Field

Only changes in field momentum can affect a Sp-2 mass. Unlike radiation, which can only exchange momentum along the target's Sp-2 axis, a change in the magnetic momentum field must act *in the plane* of a target Sp-2 rotation as a line integral around the circuit. However, in the derivation of the Coulomb interaction ([2], Sec. 4.1), even though the proton's Sp-3 momentum field acts along the poloidal axis in the plane of the Sp-3 rotation, the integration is still performed around the Sp-2 circuit, perpendicular to that rotation, in time period T_2 . A more consistent derivation should have taken an integral of the Sp-3 field momentum around the electron Sp-3 circuit in time period T_3 since the Sp-3 field momentum acts *in the plane* of that rotation. This produces the same result. It is the Sp-3 energy change due to the cumulative changing Coulomb (electric momentum) field from infinity that displaces energy out of the electron's Sp-3 rotation.

In the hydrogen atom hyperfine interaction ([21], Eq. (36)), the proton's magnetic momentum field changes due to the proton–electron relative motion and it acts on the electron's Sp-2 rotation circuit. However, the field must have a gradient across the Sp-2 circuit otherwise there is no effect. This results in either an overload or underload of Sp-2 azimuthal action which becomes displaced

along its axis and either adds to or subtracts from the electron's Sp-3 toroidal energy. We shall see in our derivation of the Lorentz force that it is this effect that produces the desired deflection.

3.5. Nature of the B Field

The mapping of magnetic field lines from a magnetic north pole involves recording the strength and direction a magnetic needle takes as its position is changed in the vicinity of the pole. In 1852, Faraday confessed that the term '[B] line of force' did not include "any idea of the nature of the physical cause of the phenomena" and that "how the magnetic force is transferred through bodies or through space we know not" [30]. However, his preference was for the transmission of an influence rather than an action at a distance.

Consider the emission of magnetic momentum field lines p_{2f} with initial azimuthal speed $c(1 - \beta^2)^{1/2}$, Figure 4b. Since a solenoid has electrons travelling in the opposite direction to the current I , then the B field lines in Figure 5b, according to the PTV model [2] must be identical with the spiral emission towards point B in Figures 4c and 4f. We note that it is the rotated radial emission direction that points towards B. The full field momentum p_{2f} is at 45° to line AB, Figure 4g.

We might ask the following about the magnetic field around a conductor in Figure 5b. If an electron is a Sp-3 structure that moves along its axis, how does the direction of its p_{2f} spiral momentum field whose axis of rotation is always perpendicular to the Sp-3 axis produce a magnetic momentum field rotation around the Sp-3 axis? Our proposed solution in relation to the model presented here, is that microscopically a magnetic momentum field is not responsible for the field around a conducting wire. When a potential difference is placed across a conducting wire, the proton Sp-3 axes align with the conductor and it is their Sp-3 electric momentum fields that rotate around the conductor. However, it does so in a spiral like the magnetic momentum field, Figure 4f. The stronger the potential difference, the more protons align. So, macroscopically the magnetic field in Ampère's law is an *electric* momentum field and the field lines do not adopt a closed curve (which would be a purely azimuthal field) but a spiral.

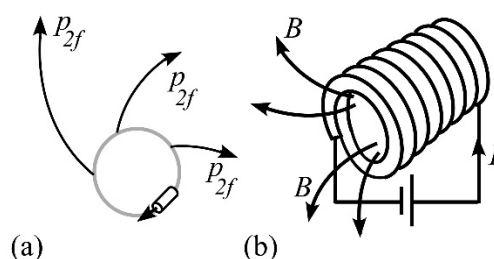


Figure 5. **a** Sp-2 circuit with emitted spiral magnetic momentum field p_{2f} . **b** Solenoid in which the electrons are moving with surrounding B field lines.

4. The Lorentz Deflection

Consider the face of a magnetic north pole N set in the $y - z$ plane. Let the radial field emission component $p_{2f}/\sqrt{2}$ in Figure 4g be perpendicular to this plane along the positive x axis. The p_{2f} must be 45° to these lines, and if we assume they are randomly orientated and the components of \vec{p}_{2f} perpendicular to these lines (around the x axis) cancel out, each emission line still has a component $p_{2f}/\sqrt{2}$ along the x direction. Also, let us imagine a single Sp-2 circuit, running in the $x - y$ plane, to be placed in this field, see Figure 6a. The field momentum magnitude is inversely proportional to the perpendicular distance r_{2f} from the plane, so that in Cartesian coordinates ([2], Eq. (21)) and referring to Figure 6b we have the effective magnetic momentum as

$$\vec{p}_{eff} = \frac{\vec{p}_{2f}}{\sqrt{2}} = \frac{\hbar}{\sqrt{2}r_{2f}} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad d\hat{r} = d\gamma \begin{pmatrix} -\sin \gamma \\ \cos \gamma \\ 0 \end{pmatrix} \quad (7)$$

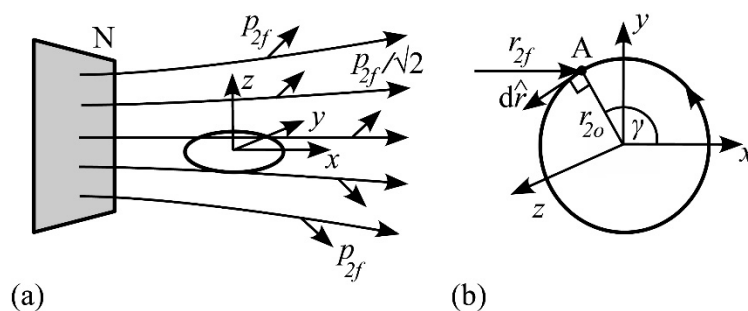


Figure 6. **a** Face of a magnetic north pole N in the $y - z$ plane emitting curved p_{2f} lines. The rotating radial direction has field momentum $p_{2f}/\sqrt{2}$ which we take along the x axis, see Figure 4g. The rotating azimuthal component is randomly orientated and so the resultant p_{2f} is also. There is also a Sp-2 circuit shown rotating in the $x - y$ plane. **b** The distance of a point A on the Sp-2 circuit from the magnetic pole N is r_{2f} , taken along the x axis perpendicular to the pole. The Sp-2 radius is r_{2o} and the angle of rotation of point A is γ . The vector $d\hat{r}$ is tangential to the circuit at A.

We now construct a Sp-3 toroid by allowing the Sp-2 circuit to rotate about the y axis through an angle τ at Sp-3 radius r_{3o} , see Figure 7 ([2], Figure 9a). Let us locate the center of rotation of the Sp-3 toroid O a perpendicular distance d from the magnetic north pole N. Then, since we are only interested in the distance of a point A from the magnetic pole N along the x axis (Figure 6b) we have

$$r_{2f} = d + (r_{3o} + r_{2o} \cos \gamma) \cos \tau \tag{8}$$

Using Eq. (7) we also have

$$d\hat{r} = dy \begin{pmatrix} -\sin \gamma \cos \tau \\ \cos \gamma \\ -\sin \gamma \sin \tau \end{pmatrix}, \quad \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \cos \tau & 0 & -\sin \tau \\ 0 & 1 & 0 \\ \sin \tau & 0 & \cos \tau \end{pmatrix} \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} \tag{9}$$

where the first equation has made use of the given rotation matrix about the y axis. The next stage is to introduce the velocity v of the Sp-3 toroid that is directed along the normal to the toroidal plane and lies in the $x - y$ plane. In Figure 8, a Sp-3 toroid is shown rotated about the z axis so that the direction of the velocity v also rotates. If the angle that the x' axis makes with the x axis is σ then the velocity vector runs along the y' axis and makes an angle $(\sigma + \pi/2)$ to the x axis.

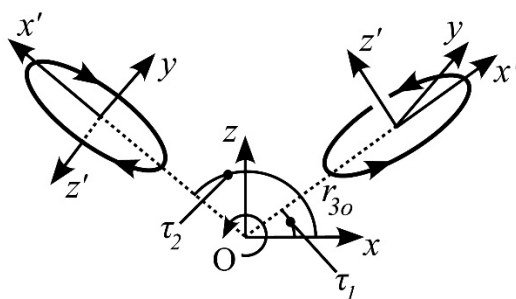


Figure 7. Sp-3 toroid of radius r_{3o} consisting of a Sp-2 circuit rotating about the y axis (not to scale). The radius r_{3o} from O to the Sp-2 center is shown in two positions at angles τ_1 and τ_2 in the $x - z$ plane. Consequently, with the y axis having a constant orientation, the x and z axes become rotated in the $x - z$ plane to x' and z' , respectively.

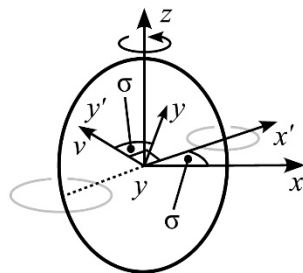


Figure 8. Sp-3 toroid initially in the $x - z$ plane rotated an angle σ about the z axis to finish in the $x' - z$ plane. The velocity v is along y' .

As stated, the \vec{p}_{2f} vectors are randomly orientated at 45° to the magnetic field lines (see Figure 6a) so the components perpendicular these lines tend to cancel out.¹⁶ However, the components parallel to the lines along the x axis reinforce so this shall be the focus our attention. The introduction of Sp-3 velocity v in relation to the momentum components that run along the x axis has the effect of producing an additional relative-to-electron momentum change in the $x - y$ plane with components parallel to the x axis and y axis. For example, if an electron has a y component of $v \cos \sigma$ perpendicular to and approaching the field line then the field change is towards the electron with an opposite sign, see Figure 9. The same argument applies to the x component. These changes in momentum will serve as the basis for the Lorentz force calculation. We take the speed of the source Sp-2 field component parallel to a field line (rotating radial component) to be $c' = c(1 - \alpha^2)^{1/2} r_{2o}/d$, see Figure 4b and Ref. ([2], Eqs.(25) and (28)) with $d = r_{2f}$.

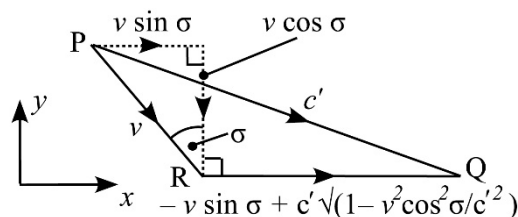


Figure 9. The motion of the magnetic momentum field line PQ perpendicular to the magnetic pole in the Sp-3 frame due to speed v through the field at an angle σ to the y axis. This creates a change in field momentum relative to the Sp-3 center in the $x - y$ plane, where c' is the speed of the magnetic momentum field component along the field line, see Eq. (10).

Figure 9 shows the magnetic momentum field passing along PQ in the Sp-3 frame as a result of the Sp-3 speed along RP. Relative to the Sp-3 frame, the field line advances along PR at speed v at an angle σ to the y axis. This has the effect of changing the field speed along the x axis in the Sp-3 frame to RQ. The field-line speed of advance along the x and y axes in that frame become $v \sin \sigma$ and $-v \cos \sigma$, respectively. We carry over the field speed ratios to the change-in-momentum ratios so, for example, $c' : v \sin \sigma = 1 : v \sin \sigma / c' = (p_{2f}/\sqrt{2}) : \Delta p_x$, see the first part of Eq. (10). Then applying a rotation about the z axis using the first part of Eq. (9), we now have the magnetic momentum field change relative to the Sp-3 construction as

¹⁶ We take the Sp-2 source circuits to have a range of orientations.

$$\Delta \vec{p}_{2f} = \frac{\hbar}{\sqrt{2}r_{2f}} \begin{pmatrix} \frac{v \sin \sigma}{c'} \\ v \cos \sigma \\ -\frac{c'}{0} \end{pmatrix}, \quad d\hat{r} = d\gamma \begin{pmatrix} -\cos \sigma \cos \tau \sin \gamma - \sin \sigma \cos \gamma \\ -\sin \sigma \cos \tau \sin \gamma + \cos \sigma \cos \gamma \\ -\sin \gamma \sin \tau \end{pmatrix},$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \cos \sigma & -\sin \sigma & 0 \\ \sin \sigma & \cos \sigma & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} \quad (10)$$

where the middle part of Eq. (10) has made use of the last part which is the rotation matrix about the z axis.

We now consider the coordinates of point A in Figure 6b under rotation transformations. First, we initialize the Sp-2 circuit at $\tau = 0$ in Eq. (8) and Figure 7, then apply a y rotation of the circuit followed by a z rotation relative to the Sp-3 center O as follows:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \cos \sigma \cos \tau & -\sin \sigma & -\cos \sigma \sin \tau \\ \sin \sigma \cos \tau & \cos \sigma & -\sin \sigma \sin \tau \\ \sin \tau & 0 & \cos \tau \end{pmatrix} \begin{pmatrix} r_{3o} + r_{2o} \cos \gamma \\ r_{2o} \sin \gamma \\ 0 \end{pmatrix} \quad (11)$$

Since we are only interested in the perpendicular distance of point A from the magnetic pole along the x axis, then after recalling that the distance to the Sp-3 center O is d , Eq. (11) leads to

$$(r_{2f})^{-1} = \frac{1}{d} \left(1 + \frac{(r_{3o} + r_{2o} \cos \gamma) \cos \sigma \cos \tau}{d} - \frac{r_{2o} \sin \sigma \sin \gamma}{d} \right)^{-1} \quad (12)$$

The calculation for the mean change in momentum around a single Sp-2 circuit taken over all Sp-3 angles τ is

$$\Delta p = \frac{N}{2\pi} \int_0^{2\pi} \vec{p}_{2f} \cdot d\hat{r} \quad (13)$$

where N is the number of discrete field hits on the Sp-3 circuit in the rest time period $T_3 = h(1 - \alpha^2)^{1/2} / (m_o \alpha^2 c^2)$ for small speeds $\beta \ll 1$ ([2], Eq. (13)). Using Eq. (10) and a second order binomial approximation to Eq. (12) we arrive at

$$\Delta p = \frac{N \hbar r_{2o} |v| \sin \sigma' \cos \tau}{2\sqrt{2} c' d^2} \left(1 - \frac{2r_{3o} \sin \sigma' \cos \tau}{d} + \dots \right) \quad (14)$$

where $\sigma' = \sigma + \pi/2$ is the angle that the velocity vector makes with the x axis in the $x - y$ plane, see Figure 9. This momentum change around a Sp-2 circuit arises from the change in field momentum perpendicular to the original field direction along the x axis, and is generated by the relative speed of the Sp-3 circuit and the field line. It is worth remarking that without a magnetic momentum field gradient across a Sp-2 circuit $\Delta p = 0$.

5. Conservation of the Speed of Light

The vector diagram in Figure 10a shows a Sp-2 circuit whose normal is rotating at speed v_3 around the Sp-3 center (not shown). When the Sp-3 structure is stationary in a field-free space then the rotation occurs at speed $v_3 = \alpha c$, and $v_f = v_n = v_a = 0$, where α is the fine structure constant. The Sp-2 speed of revolution is $v_2 = c(1 - \alpha^2)^{1/2}$. The Sp-2 normal remains perpendicular to the Sp-3 normal even when the toroid is in motion, see Ref. ([2], Section 5, Part (3)). This means that there is no need to account for the rake angle of the Sp-3 rotational trajectory: the axis of the Sp-2 rotation is always perpendicular to the Sp-3 normal velocity vector \vec{v}_n . Figure 10b shows how these vectors add in a field-free space to maintain the speed of light magnitude $|\vec{c}|$. When $|\vec{v}_n| > 0$ there must be a decrease in the mean Sp-2 speed of rotation $v_2 < c(1 - \alpha^2)^{1/2}$. In Figure 10c, the toroid now passes through magnetic field lines, and due to a redistribution of Sp-2 action (when the change-in-field interacts with the Sp-2 circuit), an additional speed v_f is created parallel (or anti-parallel) to the Sp-2 normal along v_3 . An adjustment velocity v_a is subsequently required to restore the speed of light magnitude. This whole process takes place in the Sp-2 rotation period T_2 .

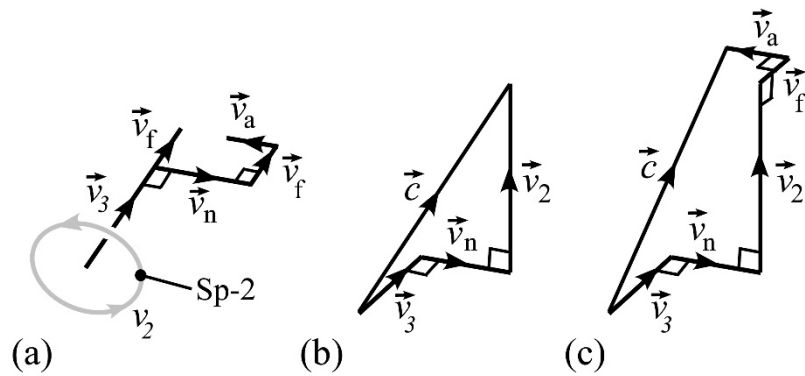


Figure 10. **a** Sp-2 circuit inside a Sp-3 structure with velocity \vec{v}_3 along the Sp-2 normal. Vector \vec{v}_n is the Sp-3 normal velocity, \vec{v}_f is the Sp-2 circuit's response to the field altering its action, and \vec{v}_a is the adjustment velocity that preserves the speed of light. All four vectors are coplanar. **b** Vector diagram in a field free space that maintains the speed of light $|\vec{c}|$. The \vec{v}_2 is the mean Sp-2 speed resulting from the introduction of \vec{v}_n . **c** Vector diagram for motion in a magnetic momentum field. To part **b** is now added the field effect \vec{v}_f on the Sp-2 circuit normal, and the light-speed restoring adjustment \vec{v}_a .

Let us first consider motion through a field-free space. If we set the toroid in motion with initial speed along the $v_n = v$ along the toroidal axis, then while the initial v_3 component remains constant, in order to keep the speed of light constant, the mean Sp-2 rotational speed v_2 must be given by

$$v_2 = c \left(1 - \frac{v_3^2 + v_n^2}{c^2} \right)^{1/2} \quad (15)$$

where $v_3 = \alpha c$. This is the v_2 component that is perpendicular to both \vec{v}_3 and \vec{v}_n , see Figure 10b.¹⁷ As we shall see, the index k will enable an iterative calculation to be performed once field effects are introduced.

We now pass the toroid through magnetic field lines set across its path. The mean Sp-2 rotational action is either overloaded or underloaded, and the gain or loss of momentum is redistributed along the direction of \vec{v}_3 as \vec{v}_f , being a velocity resulting from the field interaction. This occurs in Sp-2 time period T_2 . The calculation uses Eq. (14), where for relativistic electron mass m we have

$$v_f = \frac{\Delta p}{m} = A(\sigma', \tau) \frac{|v_n|}{m}, \quad A(\sigma', \tau) = \frac{N\hbar r_{2o} \sin \sigma' \cos \tau}{2\sqrt{2}c'd^2} \left(1 - \frac{2r_{3o} \sin \sigma' \cos \tau}{d} + \dots \right) \quad (16)$$

We note that for an electron with Sp-2 radius $r_{2o} = 3.862 \times 10^{-13} m$, $r_{3o} = 5.291 \times 10^{-11} m$, $h = 6.626 \times 10^{-34} \text{ Js}$, $c = 3 \times 10^8 \text{ ms}^{-1}$, $d = 0.01 \text{ m}$ then $A \sim N(3 \times 10^{-18})$ ([2], Eq. (1)). Equation (15) now needs adjusting so that

$$c^2 = (v_3 + v_f)^2 + (v_n + v_a)^2 + v_2^2 \quad (17)$$

Combining Eqs. (15) and (17) we obtain a quadratic in v_a that only requires the input of the translational speed v_n and the Sp-3 rotational speed v_3 :

$$v_a = -v_n \left(1 - \sqrt{1 - \frac{v_f^2}{v_n^2} - 2 \frac{v_3 v_f}{v_n^2}} \right) \sim -A \frac{v_3}{v_n} - \frac{A^2}{2} \left(1 + \frac{v_3^2}{v_n^2} \right) \quad (18)$$

¹⁷ The components parallel and antiparallel to \vec{v}_n are proportional to $v_2 = c(1 - \alpha^2)^{1/2} \pm v_n$, and they cancel out when summing the incremental magnitudes around the Sp-2 circuit. In contrast, the components perpendicular to \vec{v}_n have no $\pm v_n$ but a reduction factor $\left(1 - (v_n/c)^2 \right)^{1/2}$. With a decrease in mean Sp-2 speed we should expect the mean Sp-2 radius to increase to preserve angular momentum.

We then adjust the speed of the Sp-3 circuit along its original direction through the field as follows:

$$v_n \rightarrow v_n + v_a \sim v_n \left(1 - \frac{Av_3}{v_n} - \frac{A^2}{2} \left(1 + \frac{v_3^2}{v_n^2} \right) \right) \quad (19)$$

Since $v_n = v$ as the speed the toroid enters the field and the Sp-3 speed as v_3 , we can ensure that the speed of light in the toroidal circuit remains a constant.

Average Taken over All Sp-2 Locations

We now calculate the mean momentum change along the x axis and z axis, see Figure 7. Each location of the Sp-2 circuit center as it travels around the Sp-3 circuit is characterized by a value of τ .

(a) Equation (14) shows that when $-\pi/2 < \tau < \pi/2$ we have $\Delta p > 0$ ($0 < \sigma' < \pi$) and the Sp-2 circuit increases in momentum along its normal. This is to be redistributed into v_i (see Figure 10) which is tangential to the Sp-3 rotation.

(b) For $\pi/2 < \tau < 3\pi/2$ we find $\Delta p < 0$ so that the Sp-2 circuit loses momentum. This amounts to a gain in momentum in the opposite direction.

Let us consider Eq. (14) with the adjustment in Eq. (19). The Sp-3 normal lies along y' , see Figure 8. Projecting the vector v_i (which lies along the z' axis in Figure 7) onto the x axis with v_n in Eq. (14) replaced by v_n from Eq. (19), we find

$$\Delta p_x = -\frac{1}{\pi} \int_{-\pi/2}^{\pi/2} \Delta p \sin \tau \, d\tau = 0 \quad (20)$$

From the discussion in (a) and (b), the direction of momentum change in the z direction is the same for a point at $-z$ as it is for one at $+z$, see Figure 7. So the mean Δp_z is given by

$$\Delta p_z = \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} \Delta p \cos \tau \, d\tau \sim \frac{N\hbar r_{2o} |v_n| \sin \sigma'}{4\sqrt{2}c'd^2} \left(1 - \frac{16r_{3o} \sin \sigma'}{3\pi d} + \dots \right) \quad (21)$$

Since $A(\sigma', \tau) \sim N(8 \times 10^{-22})$ then for a relatively weak field strength — determined by N , the number of discrete field hits in time T_2 — that satisfies $A(\sigma', \tau) \ll 1$ our approximations will hold. Eq. (21) applies to a Sp-2 clockwise rotation when looking along the z' axis, combined with a Sp-3 counter clockwise rotation when viewed along the y axis, see Figure 7. This case is shown in Figure 11a. The momentum in the positive z direction in Eq. (21) mirrors the behaviour of a negative charge moving through a magnetic field.

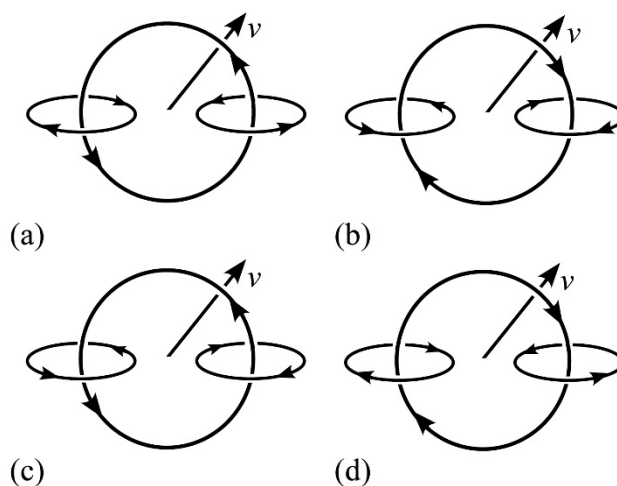


Figure 11. The four cases of charge chirality viewed in the Sp-3 direction of motion. **a** Negative charge $Q_2 = +1, Q_3 = +1$. **b** Negative charge $Q_2 = -1, Q_3 = -1$. **c** Positive charge $Q_2 = -1, Q_3 = +1$. **d** Positive charge $Q_2 = +1, Q_3 = -1$.

6. The Nature of Charge

For $\beta \ll 1$, we find the change in momentum in Eq. (21) using the Sp-3 time period $T_3 = h(1 - \alpha^2)^{1/2}/(m_o \alpha^2 c^2)$ of a Sp-3 circuit at rest ([2], Eq. (13)). Dividing Eq. (21) by T_3 gives a force

$$F_z = \frac{\Delta p_z}{T_3} = Q_2 Q_3 \frac{\sqrt{2} N m_o \alpha^2 c r_{2o} |v_n| \sin \sigma'}{16\pi(1 - \alpha^2)d^2} \quad (22)$$

where r_{2o} is the Sp-2 radius, $\beta = v/c$, while keeping in view that N is the field-absorption count in time T_3 . Here, $Q_2 = +1$ when the Sp-2 rotation is clockwise viewed in the positive z direction at $\tau = 0$, and $Q_2 = -1$ when counter clockwise; and $Q_3 = +1$ when the Sp-3 rotation is counter-clockwise viewed in the positive y direction, and $Q_3 = -1$ when clockwise, see Figures 7 and 11. Changing the direction of the Sp-3 rotation means that an action overload in the Sp-2 circuit increases the speed of its normal in the opposite direction. When $Q_2 Q_3 = +1$, the Sp-3 is deflected in the positive z direction thereby acting as a negative charge would. Conversely, for $Q_2 Q_3 = -1$ it moves in the negative z direction and acts like a positive charge. This means there are two chiralities that can be associated with a negative charge and two for a positive one, see Figure 11.

From Ref. ([2], Eqs. (63) and (70)) the Sp-2 radius is

$$r_{2o} = \frac{\hbar}{m_o c} \quad (24)$$

It is now only a small step to generalize to the Lorentz force so that a comparison with Eq. (1) suggests that

$$q|\vec{B}| = Q_2 Q_3 \frac{\sqrt{2} N m_o \alpha^2 c r_{2o} |v_n| \sin \sigma'}{16\pi(1 - \alpha^2)d^2} \quad (25)$$

A calibration of N with $|\vec{B}|$ for various d should then be possible from experiment. It should also be noted that if the speed v in Figure 11a is reversed, as might occur if a reference frame carrying the magnetic pole accelerates past it, we arrive at the arrangement in Figure 11b which is classed here as the same charge.¹⁸ However, since $\sigma' \rightarrow \sigma' + \pi$ in Eq. (22), the direction of deflection is also reversed.

7. Electric Fields

Figure 12a shows a bound electron (right) and proton (left). In virtue of the proton's electric momentum field (Coulomb field) the electron Sp-3 is underloaded. This means half of its Sp-3 energy has been redistributed into translational energy that has been radiated away as ionization energy. It then stops at the bound state distance from the proton. If the electron is now reloaded exactly up to its capacity by a photon rotating in the same sense as the electron Sp-3, then it should relocate to infinity where it originated from and stop. If its action is overloaded, then it should acquire more than the required energy to take it there. The main point is that in being ionized, the direction of motion of the electron is reversed as shown in Figure 12b. In Figure 12a the proton is Figure 11d. For the electron, we have the case from Figure 11b, while in Figure 12b it has transformed into the case in Figure 11a. It should also be noted that for the ionization in Figure 12a, the proton and electron have same-sense Sp-2 rotations which is the combination of least action.

¹⁸ When reversing the speed direction of the Sp-3, the view of Fig. 11a from the right side becomes the view of Fig. 11b from the left side.

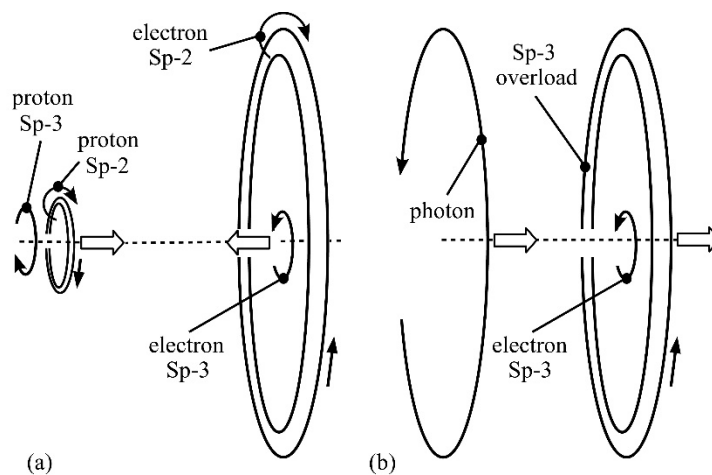


Figure 12. **a** The bound state of the proton and electron with same-sense Sp-2 rotations. **b** A circularly polarized photon with the same rotation sense as the electron impinges on it and overloads its action. Consequently, the electron takes on a repulsion configuration (active acceleration) and reverses direction in relation to which the rotation sense becomes counter clockwise.

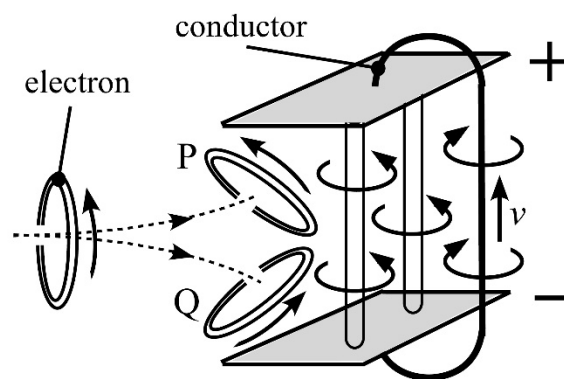


Figure 13. Ionized electron from Figure 12b approaching two parallel plates. The conductor connecting the two plates has a counter clockwise momentum field in the direction of the electron velocity v . The continuity of field rotation means that the electric field tubes between the plates rotate clockwise moving from negative to positive. If the electron P approaches the upper plate, the electron and field rotations oppose and there is attraction. At Q, the rotations are same sense and constitute a repulsion.

We now suppose that when a potential difference is applied across a conducting wire, the motion of the electrons as Figure 11a along the conductor from the negative to the positive plate causes their toroidal axes to align. Consequently, their electric field momenta rotate around the wire counter clockwise when viewed in their direction of motion. The electric field tubes between the plates continue this rotation, being counter clockwise from the positive to the negative plate. The Sp-2 axes of the electrons remain perpendicular to the toroidal axes and play no part in the circulating magnetic field. This means that when the ionized electron in Figure 12b approaches the positive plate their rotations oppose and cause an attraction as in the hydrogen atom. If instead it approaches the negative plate their rotations reinforce to produce a repulsion.

8. Conclusions

We have shown how it is possible to obtain the Lorentz force law of deflection from a mass represented as a Sp-2 (optical OAM). The motion of the Sp-3 configuration towards a field line set across its path generates a field momentum perpendicular to the field line. This interacts with the various orientations of the Sp-2 (optical OAM) circuit in the Sp-3 rotation of the moving mass. If the

field strength had no variation along the x axis (see Figure 6a), there would be no net change-in-momentum field (generated by the toroidal motion along the y axis) when integrated around a Sp-2 circuit. However, since the magnetic momentum varies with d , the perpendicular distance from the pole, the field strength on the magnetic pole side of the Sp-2 is different from that on the opposite side. This excess or deficit of momentum integrated around the Sp-2 circuit is then redistributed along the Sp-2 normal.

The theory presented here, based on the principles of the photonic toroidal vortex (PTV) model set out in Ref. [2], suggests a geometrical nature of charge in terms of a combination of Sp-2 and Sp-3 rotations. As far as the PTV theory of the hydrogen atom is concerned [2], it reinforces the assumption that an ‘electron’ Sp-2 orbiting in a Sp-3 toroid has two Sp-2 rotation senses, see Figures 11a and 11b. This duality has been shown to be crucial for the hydrogen hyperfine interaction [21] and lends credit to the claim of consistency in the PTV theory. It also raises the possibility that when the Sp-2 rotation reverses sense in an ‘electron’ bound in a hydrogen atom [21], it becomes a *positron*. However, if it is never ionized as a positron but only as an electron that reverts back to its original Sp-2 sense on release. This is because the electron configuration in Figure 11b inside the hydrogen atom has the same Sp-2 rotation sense as the proton in Figure 11d. This constitutes an electron Sp-2 red shift which is the lower action case of the two possible electron Sp-2 rotations. So the “stealth positron” is never observed due to structural suppression at ionization by the least action principle that favours the electron. This might account for the infrequent observation of free positrons in comparison with free electrons.

On behalf of all authors, the corresponding author states that there is no conflict of interest.

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