10:31:41 Jan 06, 2026

*** A program to calculate hydrogen atom hyperfine levels using the new photonic toroidal vortex (PTV) model ***
                written in Liberty BASIC by Dr Barry R. Clarke, completed 28 May 2024

 All equations and tables refer to the following papers.

 PTV1: Clarke, Barry R., A photonic toroidal vortex model of the hydrogen atom fine structure, Quantum Studies: Mathematics and Foundations, 12, article 18 (2025)
 PTV2: Clarke, Barry R., present hyperfine paper

 EXPERIMENTAL DATA
 Horbatsch, M. and E. A. Hessels, Tabulation of the bound-state energies of atomic hydrogen, Physical Review A 93, 022513 (2016)

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POSITIVE GAMMA (emult)

1S1/2
QUANTUM NUMBERS
(1) Quantum Mechanics: n = 1 L = 0 Sp = 0.5   Sommerfeld: n(phi) = 1 n(r) = 0

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers =   1.7324035537303600 PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers =   2.0000332836086264 PTV1 d2-bar Eq (86)

 emult =   1.0069427000

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments   =   3289885758.535002
(5) New PTV f-s energy no adjustments   =   3289885758.260552   PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass   =   3288095006.026450
(7) Relativistic reduced mass multiplier   =   0.9994556721741642   PTV2 $(1 + Mr)^{-1}$ Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   3288095029.923980   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier   =   1.0010971   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
    and adjusted velocity   =   3291702389.145564   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy   =   3288087212.213100 Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
    and average h-f energy   =   3615176.932463
(13) Exit potential   =   3615176.932463   PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude   =   0.000000
(15) H-f mid-point PTV   =   3288087212.213100

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Lower h-f energy from experiment   =   3288086502.010200   Horbatsch and Hessels (2016)
(17) Lower h-f energy calculated from PTV =   3288086502.193267   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   17961171874.889158
(19) H-f difference taken from experiment =   710.202900
(20) H-f difference calculated from PTV =   710.019833   PTV2 Eq (37)
(21) H-f error magnitude   =   0.183066

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2S1/2
QUANTUM NUMBERS
(1) Quantum Mechanics: n = 2 L = 0 Sp = 0.5   Sommerfeld: n(phi) = 1 n(r) = 1

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers =   3.8732511083690200 PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers =   4.0001231494850968 PTV1 d2-bar Eq (86)

 emult =   1.0069427000

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments   =   822474177.055886
(5) New PTV f-s energy no adjustments   =   822474177.045523   PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass   =   822026487.438713
(7) Relativistic reduced mass multiplier   =   0.9994556776131092   PTV2 $(1 + Mr)^{-1}$ Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   822026493.441384   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier   =   1.00054847   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
    and adjusted velocity   =   822477353.040722   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy   =   822025488.313800 Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
    and average h-f energy   =   451864.726922
(13) Exit potential   =   451864.726922 PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude   =   0.000000
(15) H-f mid-point PTV   =   822025488.313800

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Lower h-f energy from experiment   =   822025399.535400   Horbatsch and Hessels (2016)
(17) Lower h-f energy calculated from PTV =   822025399.565678   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   27149399551.735912
(19) H-f difference taken from experiment =   88.778400
(20) H-f difference calculated from PTV =   88.748122   PTV2 Eq (37)
(21) H-f error magnitude   =   0.030278

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3S1/2
QUANTUM NUMBERS
(1) Quantum Mechanics: n = 3 L = 0 Sp = 0.5   Sommerfeld: n(phi) = 1 n(r) = 2

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers =   5.9163991001174104 PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers =   6.0002241103507824 PTV1 d2-bar Eq (86)

 emult =   1.0069427000

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments   =   365542862.051824
(5) New PTV f-s energy no adjustments   =   365542862.054546   PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass   =   365343889.550878
(7) Relativistic reduced mass multiplier   =   0.9994556786197338   PTV2 $(1 + Mr)^{-1}$ Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   365343891.037741   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier   =   1.00036563   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
    and adjusted velocity   =   365477472.518026   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy   =   365343591.599600 Horbatsch and Hessels (2016)

(12) Difference between PTV f-s with corrections
    and average h-f energy   =   133880.918426
(13) Exit potential   =   133880.918426   PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude   =   0.000000
(15) H-f mid-point PTV   =   365343591.599600

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Lower h-f energy from experiment   =   365343565.294900   Horbatsch and Hessels (2016)
(17) Lower h-f energy calculated from PTV =   365343565.304514   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   38001239393.181736
(19) H-f difference taken from experiment =   26.304700
(20) H-f difference calculated from PTV =   26.295086   PTV2 Eq (37)
(21) H-f error magnitude   =   0.009614

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4S1/2
QUANTUM NUMBERS
(1) Quantum Mechanics: n = 4 L = 0 Sp = 0.5   Sommerfeld: n(phi) = 1 n(r) = 3

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers =   7.9376529632206608   PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers   =   8.0003278449755088   PTV1 d2-bar Eq (86)

 emult =   1.0069427000

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments   =   205617346.644788
(5) New PTV f-s energy no adjustments   =   205617346.640405   PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass   =   205505424.892383
(7) Relativistic reduced mass multiplier   =   0.9994556789719766   PTV2 $(1 + Mr)^{-1}$ Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   205505425.404479   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier   =   1.00027422   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
    and adjusted velocity   =   205561778.663135   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy   =   205505298.855200 Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
    and average h-f energy   =   56479.807935
(13) Exit potential   =   56479.712139   PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude   =   0.095796
(15) H-f mid-point PTV   =   205505298.950996

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Lower h-f energy from experiment   =   205505287.757900   Horbatsch and Hessels (2016)
(17) Lower h-f energy calculated from PTV =   205505287.857912   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   2054802537.094273
(19) H-f difference taken from experiment =   11.097300
(20) H-f difference calculated from PTV =   11.093084   PTV2 Eq (37)
(21) H-f error magnitude   =   0.004216

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5S1/2
QUANTUM NUMBERS
(1) Quantum Mechanics: n = 5 L = 0 Sp = 0.5   Sommerfeld: n(phi) = 1 n(r) = 4

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers =   9.9503639585056512   PTV1 x-bar Eqs (84)/(86)

(3) Shortest distance between p-e Sp-2 centers   =   10.0004326891036560   PTV1 d2-bar Eq (86)

 emult =   1.0069427000

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments   =   131594869.718334
(5) New PTV f-s energy no adjustments   =   131594869.717838   PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass   =   131523239.923151
(7) Relativistic reduced mass multiplier   =   0.9994556791349960   PTV2 $(1 + Mr)^{\wedge}(-1)$ Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   131523240.145725   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier   =   1.00021937   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
    and adjusted velocity   =   131552092.567694   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy   =   131523175.306400   Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
    and average h-f energy   =   28917.261294
(13) Exit potential   =   28917.141892   PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude   =   0.119401
(15) H-f mid-point PTV   =   131523175.425801

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Lower h-f energy from experiment   =   131523169.624600   Horbatsch and Hessels (2016)
(17) Lower h-f energy calculated from PTV   =   131523169.746193   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   1081665040.215909
(19) H-f difference taken from experiment   =   5.681800
(20) H-f difference calculated from PTV   =   5.679608   PTV2 Eq (37)
(21) H-f error magnitude   =   0.002192

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6S1/2
QUANTUM NUMBERS
(1) Quantum Mechanics: n = 6 L = 0 Sp = 0.5   Sommerfeld: n(phi) = 1 n(r) = 5

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers   =   11.9588462377676944   PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers   =   12.0005380879831920   PTV1 d2-bar Eq (86)

 emult =   1.0069427000

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments   =   91385208.595505
(5) New PTV f-s energy no adjustments   =   91385208.586172   PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass   =   91335465.746194
(7) Relativistic reduced mass multiplier   =   0.9994556792235434   PTV2 $(1 + Mr)^{\wedge}(-1)$ Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   91335465.848147   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier   =   1.00018281   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
    and adjusted velocity   =   91352162.677503   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy   =   91335428.313650   Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
    and average h-f energy   =   16734.363853
(13) Exit potential   =   16734.249498   PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude   =   0.114355
(15) H-f mid-point PTV   =   91335428.428005

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Lower h-f energy from experiment   =   91335425.025600   Horbatsch and Hessels (2016)
(17) Lower h-f energy calculated from PTV =   91335425.141215   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   789995643.194370
(19) H-f difference taken from experiment =   3.288050
(20) H-f difference calculated from PTV   =   3.286790   PTV2 Eq (37)
(21) H-f error magnitude   =   0.001260

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2P1/2
QUANTUM NUMBERS
(1) Quantum Mechanics: n = 2 L = 1 Sp = -0.5   Sommerfeld: n(phi) = 1 n(r) = 1

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers =   3.8732511083690200   PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers   =   4.0001231494850968   PTV1 d2-bar Eq (86)

 emult =   0.5577389026

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments   =   822474177.055886
(5) New PTV f-s energy no adjustments   =   822474177.045523   PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass   =   822026487.438713
(7) Relativistic reduced mass multiplier   =   0.9994556776135524   PTV2 (1 + Mr)^(-1) Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   822026490.094327   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier   =   1.00030378   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
    and adjusted velocity   =   822276202.972314   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy   =   822026516.551150   Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
    and average h-f energy   =   249686.421163
(13) Exit potential   =   249686.421164   PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude   =   0.000000
(15) H-f mid-point PTV   =   822026516.551150

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Lower h-f energy from experiment   =   822026486.966400   Horbatsch and Hessels (2016)
(17) Lower h-f energy calculated from PTV =   822026486.968443   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   402478139542.300800
(19) H-f difference taken from experiment =   29.584750
(20) H-f difference calculated from PTV   =   29.582707   PTV2 Eq (37)
(21) H-f error magnitude   =   0.002043

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3P1/2
QUANTUM NUMBERS
(1) Quantum Mechanics: n = 3 L = 1 Sp = -0.5   Sommerfeld: n(phi) = 1 n(r) = 2

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers =   5.9163991001174104   PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers   =   6.0002241103507824   PTV1 d2-bar Eq (86)

 emult =   0.5577389026

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments   =   365542862.051824

(5) New PTV f-s energy no adjustments   =   365542862.054546   PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass   =   365343889.550878
(7) Relativistic reduced mass multiplier   =   0.9994556786198650   PTV2 $(1 + Mr)^{-1}$ Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   365343890.244386   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier   =   1.00020251   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
    and adjusted velocity   =   365417877.126191   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy   =   365343897.705100   Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
    and average h-f energy   =   73979.421091
(13) Exit potential   =   73979.421091   PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude   =   0.000000
(15) H-f mid-point PTV   =   365343897.705100

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Lower h-f energy from experiment   =   365343888.939200   Horbatsch and Hessels (2016)
(17) Lower h-f energy calculated from PTV =   365343888.940071   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   419308615338.142656
(19) H-f difference taken from experiment =   8.765900
(20) H-f difference calculated from PTV   =   8.765029   PTV2 Eq (37)
(21) H-f error magnitude   =   0.000871

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4P1/2
QUANTUM NUMBERS
(1) Quantum Mechanics: n = 4 L = 1 Sp = -0.5   Sommerfeld: n(phi) = 1 n(r) = 3

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers =   7.9376529632206608   PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers =   8.0003278449755088   PTV1 d2-bar Eq (86)

 emult =   0.5577389026

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments   =   205617346.644788
(5) New PTV f-s energy no adjustments   =   205617346.640405   PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass   =   205505424.892383
(7) Relativistic reduced mass multiplier   =   0.9994556789720318   PTV2 $(1 + Mr)^{-1}$ Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   205505425.132543   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier   =   1.00015188   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
    and adjusted velocity   =   205536637.875342   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy   =   205505428.231800   Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
    and average h-f energy   =   31209.643542
(13) Exit potential   =   31209.643542   PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude   =   0.000000
(15) H-f mid-point PTV   =   205505428.231800

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Lower h-f energy from experiment   =   205505424.533700   Horbatsch and Hessels (2016)
(17) Lower h-f energy calculated from PTV =   205505424.534105   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   506956167706.746688
(19) H-f difference taken from experiment =   3.698100
(20) H-f difference calculated from PTV   =   3.697695   PTV2 Eq (37)

(21) H-f error magnitude   =   0.000405

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5P1/2
QUANTUM NUMBERS
(1) Quantum Mechanics: n = 5 L = 1 Sp = -0.5   Sommerfeld: n(phi) = 1 n(r) = 4

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers =   9.9503639585056512   PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers   =   10.0004326891036560 PTV1 d2-bar Eq (86)

 emult =   0.5577389026

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments   =   131594869.718334
(5) New PTV f-s energy no adjustments   =   131594869.717838   PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass   =   131523239.923151
(7) Relativistic reduced mass multiplier   =   0.9994556791350242   PTV2 $(1 + Mr)^{-1}$ Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   131523240.029201   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier   =   1.00012151   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
     and adjusted velocity   =   131539220.803966   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy   =   131523241.607050   Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
     and average h-f energy   =   15979.196916
(13) Exit potential   =   15979.194057   PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude   =   0.002859
(15) H-f mid-point PTV   =   131523241.609909

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Lower h-f energy from experiment   =   131523239.713600   Horbatsch and Hessels (2016)
(17) Lower h-f energy calculated from PTV =   131523239.716706   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   42346139338.012904
(19) H-f difference taken from experiment =   1.893450
(20) H-f difference calculated from PTV   =   1.893203   PTV2 Eq (37)
(21) H-f error magnitude   =   0.000247

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6P1/2
QUANTUM NUMBERS
(1) Quantum Mechanics: n = 6 L = 1 Sp = -0.5   Sommerfeld: n(phi) = 1 n(r) = 5

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers =   11.9588462377676944 PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers   =   12.0005380879831920 PTV1 d2-bar Eq (86)

 emult =   0.5577389026

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments   =   91385208.595505
(5) New PTV f-s energy no adjustments   =   91385208.586172   PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass   =   91335465.746194
(7) Relativistic reduced mass multiplier   =   0.9994556792235596   PTV2 $(1 + Mr)^{-1}$ Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   91335465.790301   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier   =   1.00010125   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass

and adjusted velocity   =   91344713.865123   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy   =   91335466.701500   Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
     and average h-f energy   =   9247.163623
(13) Exit potential   =   9247.159226   PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude   =   0.004397
(15) H-f mid-point PTV   =   91335466.705897

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Lower h-f energy from experiment   =   91335465.605800   Horbatsch and Hessels (2016)
(17) Lower h-f energy calculated from PTV =   91335465.610300   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   20294948098.008756
(19) H-f difference taken from experiment =   1.095700
(20) H-f difference calculated from PTV   =   1.095597   PTV2 Eq (37)
(21) H-f error magnitude   =   0.000103

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7P1/2
QUANTUM NUMBERS
(1) Quantum Mechanics: n = 7 L = 1 Sp = -0.5   Sommerfeld: n(phi) = 1 n(r) = 6

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers =   13.9649244845825792 PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers =   14.0006438038634048 PTV1 d2-bar Eq (86)

 emult =   0.5577389026

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments   =   67140087.867507
(5) New PTV f-s energy no adjustments   =   67140087.880927   PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass   =   67103542.136257
(7) Relativistic reduced mass multiplier   =   0.9994556792769424   PTV2 $(1 + Mr)^{-1}$ Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   67103542.179303   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier   =   1.00008679   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
     and adjusted velocity   =   67109366.013103   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy   =   67103542.752550   Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
     and average h-f energy   =   5823.260553
(13) Exit potential   =   5823.255441   PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude   =   0.005112
(15) H-f mid-point PTV   =   67103542.757662

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Lower h-f energy from experiment   =   67103542.062800   Horbatsch and Hessels (2016)
(17) Lower h-f energy calculated from PTV =   67103542.067727   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   13620660907.392432
(19) H-f difference taken from experiment =   0.689750
(20) H-f difference calculated from PTV   =   0.689935   PTV2 Eq (37)
(21) H-f error magnitude   =   0.000185

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2P3/2
QUANTUM NUMBERS

(1) Quantum Mechanics: n = 2 L = 1 Sp = 0.5   Sommerfeld: n(phi) = 2 n(r) = 0

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers =   3.8733061076133600 PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers   =   4.0001764043401488 PTV1 d2-bar Eq (86)

 emult =   0.4887853700

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments   =   822463227.394777
(5) New PTV f-s energy no adjustments   =   822463227.397312   PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass   =   822015543.737730
(7) Relativistic reduced mass multiplier   =   0.9994556776136446   PTV2 (1 + Mr)^(-1) Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   822015542.979021   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier   =   1.00026622   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
    and adjusted velocity   =   822234378.763348   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy   =   822015535.670900 Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
    and average h-f energy   =   218843.092448
(13) Exit potential   =   218843.092447 PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude   =   0.000001
(15) H-f mid-point PTV   =   822015535.670901

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Lower h-f energy from experiment   =   822015523.845100   Horbatsch and Hessels (2016)
(17) Lower h-f energy calculated from PTV =   822015523.838448   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   123574237011.006912
(19) H-f difference taken from experiment =   11.825800
(20) H-f difference calculated from PTV   =   11.832453   PTV2 Eq (37)
(21) H-f error magnitude   =   0.006653

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3P3/2
QUANTUM NUMBERS
(1) Quantum Mechanics: n = 3 L = 1 Sp = 0.5   Sommerfeld: n(phi) = 2 n(r) = 1

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers =   5.9164531096983408 PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers   =   6.0002773654076936 PTV1 d2-bar Eq (86)

 emult =   0.4887853700

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments   =   365539617.722525
(5) New PTV f-s energy no adjustments   =   365539617.710736   PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass   =   365340646.987534
(7) Relativistic reduced mass multiplier   =   0.9994556786198924   PTV2 (1 + Mr)^(-1) Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   365340646.969272   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier   =   1.00017748   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
    and adjusted velocity   =   365405485.835376   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy   =   365340644.107950 Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
    and average h-f energy   =   64841.727426

(13) Exit potential   =   64841.727426   PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude   =   0.000000
(15) H-f mid-point PTV   =   365340644.107950

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Lower h-f energy from experiment   =   365340640.604000   Horbatsch and Hessels (2016)
(17) Lower h-f energy calculated from PTV =   365340640.602063   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   188620102196.937408
(19) H-f difference taken from experiment =   3.503950
(20) H-f difference calculated from PTV   =   3.505887   PTV2 Eq (37)
(21) H-f error magnitude   =   0.001937

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4P3/2
QUANTUM NUMBERS
(1) Quantum Mechanics: n = 4 L = 1 Sp = 0.5   Sommerfeld: n(phi) = 2 n(r) = 2

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers =   7.9377066388691312   PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers   =   8.0003811001296544   PTV1 d2-bar Eq (86)

 emult =   0.4887853700

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments   =   205615977.950867
(5) New PTV f-s energy no adjustments   =   205615977.934378   PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass   =   205504056.943471
(7) Relativistic reduced mass multiplier   =   0.9994556789720434   PTV2 (1 + Mr)^(-1) Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   205504056.947704   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier   =   1.0001331   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
     and adjusted velocity   =   205531410.533370   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy   =   205504055.622150   Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
     and average h-f energy   =   27354.911220
(13) Exit potential   =   27354.911220   PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude   =   0.000000
(15) H-f mid-point PTV   =   205504055.622150

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Lower h-f energy from experiment   =   205504054.143900   Horbatsch and Hessels (2016)
(17) Lower h-f energy calculated from PTV =   205504054.143112   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   260643022773.503616
(19) H-f difference taken from experiment =   1.478250
(20) H-f difference calculated from PTV   =   1.479038   PTV2 Eq (37)
(21) H-f error magnitude   =   0.000788

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5P3/2
QUANTUM NUMBERS
(1) Quantum Mechanics: n = 5 L = 1 Sp = 0.5   Sommerfeld: n(phi) = 2 n(r) = 3

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers =   9.9504174816877120   PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers   =   10.0004859443149632   PTV1 d2-bar Eq (86)

emult =   0.4887853700

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments   =   131594168.930585
(5) New PTV f-s energy no adjustments   =   131594168.941192   PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass   =   131522539.516855
(7) Relativistic reduced mass multiplier   =   0.9994556791350302   PTV2 $(1 + Mr)^{-1}$ Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   131522539.541534   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier   =   1.00010648   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
    and adjusted velocity   =   131536544.478135   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy   =   131522538.831750   Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
    and average h-f energy   =   14005.646385
(13) Exit potential   =   14005.643826   PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude   =   0.002559
(15) H-f mid-point PTV   =   131522538.834309

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Lower h-f energy from experiment   =   131522538.074900   Horbatsch and Hessels (2016)
(17) Lower h-f energy calculated from PTV =   131522538.077045   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   61330147104.911136
(19) H-f difference taken from experiment =   0.756850
(20) H-f difference calculated from PTV   =   0.757265   PTV2 Eq (37)
(21) H-f error magnitude   =   0.000415

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6P3/2
QUANTUM NUMBERS
(1) Quantum Mechanics: n = 6 L = 1 Sp = 0.5   Sommerfeld: n(phi) = 2 n(r) = 4

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers =   11.9588996786783616   PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers   =   12.0005913432321440   PTV1 d2-bar Eq (86)

 emult =   0.4887853700

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments   =   91384803.042340
(5) New PTV f-s energy no adjustments   =   91384803.044546   PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass   =   91335060.413780
(7) Relativistic reduced mass multiplier   =   0.9994556792235632   PTV2 $(1 + Mr)^{-1}$ Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   91335060.424572   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier   =   1.00008874   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
    and adjusted velocity   =   91343165.094059   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy   =   91335060.003300   Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
    and average h-f energy   =   8105.090759
(13) Exit potential   =   8105.086892   PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude   =   0.003868
(15) H-f mid-point PTV   =   91335060.007168

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Lower h-f energy from experiment   =   91335059.565300   Horbatsch and Hessels (2016)

(17) Lower h-f energy calculated from PTV =   91335059.568937   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   25114283744.979172
(19) H-f difference taken from experiment =   0.438000
(20) H-f difference calculated from PTV =   0.438231   PTV2 Eq (37)
(21) H-f error magnitude   =   0.000231

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7P3/2
QUANTUM NUMBERS
(1) Quantum Mechanics: n = 7 L = 2 Sp = -0.5   Sommerfeld: n(phi) = 2 n(r) = 5

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers =   13.9649778760733312 PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers   =   14.00069705913896616 PTV1 d2-bar Eq (86)

 emult =   0.4887853700

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments   =   67139832.496862
(5) New PTV f-s energy no adjustments   =   67139832.496326   PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass   =   67103286.904616
(7) Relativistic reduced mass multiplier   =   0.9994556792769444   PTV2 $(1 + Mr)^{-1}$ Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   67103286.909409   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier   =   1.00007606   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
     and adjusted velocity   =   67108390.706689   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy   =   67103286.639850   Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
     and average h-f energy   =   5104.066839
(13) Exit potential   =   5104.062531   PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude   =   0.004308
(15) H-f mid-point PTV   =   67103286.644158

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Lower h-f energy from experiment   =   67103286.364000   Horbatsch and Hessels (2016)
(17) Lower h-f energy calculated from PTV =   67103286.368188   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   16022861834.387940
(19) H-f difference taken from experiment =   0.275850
(20) H-f difference calculated from PTV =   0.275970   PTV2 Eq (37)
(21) H-f error magnitude   =   0.000120

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3D3/2
QUANTUM NUMBERS
(1) Quantum Mechanics: n = 3 L = 2 Sp = -0.5   Sommerfeld: n(phi) = 2 n(r) = 1

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers =   5.9164531096983408 PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers   =   6.0002773654076936 PTV1 d2-bar Eq (86)

 emult =   0.3957755000

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments   =   365539617.722525
(5) New PTV f-s energy no adjustments   =   365539617.710736   PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass   =   365340646.987534

(7) Relativistic reduced mass multiplier   =   0.9994556786199196   PTV2 $(1 + Mr)^{-1}$ Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   365340646.914528   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier   =   1.0001437   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
    and adjusted velocity   =   365393147.295243   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy   =   365340649.089800 Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
    and average h-f energy   =   52498.205443
(13) Exit potential   =   52498.205443 PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude   =   0.000000
(15) H-f mid-point PTV   =   365340649.089800

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Lower h-f energy from experiment   =   365340646.986500   Horbatsch and Hessels (2016)
(17) Lower h-f energy calculated from PTV =   365340646.986267   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   1570432730738.472960
(19) H-f difference taken from experiment =   2.103300
(20) H-f difference calculated from PTV =   2.103532   PTV2 Eq (37)
(21) H-f error magnitude   =   0.000232

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4D3/2
QUANTUM NUMBERS
(1) Quantum Mechanics: n = 4 L = 2 Sp = -0.5   Sommerfeld: n(phi) = 2 n(r) = 2

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers =   7.9377066388691312 PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers =   8.0003811001296544 PTV1 d2-bar Eq (86)

 emult =   0.3957755000

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments   =   205615977.950867
(5) New PTV f-s energy no adjustments   =   205615977.934378   PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass   =   205504056.943471
(7) Relativistic reduced mass multiplier   =   0.9994556789720550   PTV2 $(1 + Mr)^{-1}$ Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   205504056.926051   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier   =   1.00010778   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
    and adjusted velocity   =   205526205.319114   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy   =   205504057.762150 Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
    and average h-f energy   =   22147.556964
(13) Exit potential   =   22147.556964 PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude   =   0.000000
(15) H-f mid-point PTV   =   205504057.762150

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Lower h-f energy from experiment   =   205504056.874800   Horbatsch and Hessels (2016)
(17) Lower h-f energy calculated from PTV =   205504056.874727   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   2805358788498.620928
(19) H-f difference taken from experiment =   0.887350
(20) H-f difference calculated from PTV =   0.887423   PTV2 Eq (37)
(21) H-f error magnitude   =   0.000073

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5D3/2
QUANTUM NUMBERS
(1) Quantum Mechanics: n = 5 L = 2 Sp = -0.5   Sommerfeld: n(phi) = 2 n(r) = 3

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers =   9.9504174816877120   PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers   =   10.0004859443149632   PTV1 d2-bar Eq (86)

 emult =   0.3957755000

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments   =   131594168.930585
(5) New PTV f-s energy no adjustments   =   131594168.941192   PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass   =   131522539.516855
(7) Relativistic reduced mass multiplier   =   0.9994556791350358   PTV2 $(1 + Mr)^{-1}$ Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   131522539.531601   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier   =   1.00008622   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
    and adjusted velocity   =   131533879.442838   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy   =   131522539.938050   Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
    and average h-f energy   =   11339.504788
(13) Exit potential   =   11339.504788   PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude   =   0.000000
(15) H-f mid-point PTV   =   131522539.938050

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Lower h-f energy from experiment   =   131522539.483700   Horbatsch and Hessels (2016)
(17) Lower h-f energy calculated from PTV =   131522539.483691   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   14493149778565.277696
(19) H-f difference taken from experiment =   0.454350
(20) H-f difference calculated from PTV   =   0.454359   PTV2 Eq (37)
(21) H-f error magnitude   =   0.000009

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6D3/2
QUANTUM NUMBERS
(1) Quantum Mechanics: n = 6 L = 2 Sp = -0.5   Sommerfeld: n(phi) = 2 n(r) = 4

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers =   11.9588996786783616   PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers   =   12.0005913432321440   PTV1 d2-bar Eq (86)

 emult =   0.3957755000

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments   =   91384803.042340
(5) New PTV f-s energy no adjustments   =   91384803.044546   PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass   =   91335060.413780
(7) Relativistic reduced mass multiplier   =   0.9994556792235666   PTV2 $(1 + Mr)^{-1}$ Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   91335060.419440   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier   =   1.00007185   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
    and adjusted velocity   =   91341622.841876   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy   =   91335060.647200   Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
    and average h-f energy   =   6562.194676
(13) Exit potential   =   6562.194358   PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude   =   0.000318
(15) H-f mid-point PTV   =   91335060.647518

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Lower h-f energy from experiment   =   91335060.384300   Horbatsch and Hessels (2016)
(17) Lower h-f energy calculated from PTV =   91335060.384580   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   326709245016.884864
(19) H-f difference taken from experiment =   0.262900
(20) H-f difference calculated from PTV   =   0.262939   PTV2 Eq (37)
(21) H-f error magnitude   =   0.000039

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7D3/2
QUANTUM NUMBERS
(1) Quantum Mechanics: n = 7 L = 2 Sp = -0.5   Sommerfeld: n(phi) = 2 n(r) = 5

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers =   13.9649778760733312   PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers   =   14.0006970591389616   PTV1 d2-bar Eq (86)

 emult =   0.3957755000

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments   =   67139832.496862
(5) New PTV f-s energy no adjustments   =   67139832.496326   PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass   =   67103286.904616
(7) Relativistic reduced mass multiplier   =   0.9994556792769464   PTV2 $(1 + Mr)^{-1}$ Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   67103286.906507   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier   =   1.00006159   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
    and adjusted velocity   =   67107419.498716   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy   =   67103287.046850   Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
    and average h-f energy   =   4132.451866
(13) Exit potential   =   4132.451271   PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude   =   0.000595
(15) H-f mid-point PTV   =   67103287.047445

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Lower h-f energy from experiment   =   67103286.881300   Horbatsch and Hessels (2016)
(17) Lower h-f energy calculated from PTV =   67103286.881863   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   119153435201.030496
(19) H-f difference taken from experiment =   0.165550
(20) H-f difference calculated from PTV   =   0.165582   PTV2 Eq (37)
(21) H-f error magnitude   =   0.000032

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8D3/2
QUANTUM NUMBERS
(1) Quantum Mechanics: n = 8 L = 2 Sp = -0.5   Sommerfeld: n(phi) = 2 n(r) = 6

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers =   15.9695580628726016   PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers =   16.0008029731643200   PTV1 d2-bar Eq (86)

 emult =   0.3957755000

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments   =   51403919.639947
(5) New PTV f-s energy no adjustments   =   51403919.634302   PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass   =   51375939.428839
(7) Relativistic reduced mass multiplier   =   0.9994556793115918   PTV2 (1 + Mr)^(-1) Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   51375939.424866   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier   =   1.00005389   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
    and adjusted velocity   =   51378707.932672   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy   =   51375939.517400   Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
    and average h-f energy   =   2768.415272
(13) Exit potential   =   2768.414488   PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude   =   0.000784
(15) H-f mid-point PTV   =   51375939.518184

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Lower h-f energy from experiment   =   51375939.406500   Horbatsch and Hessels (2016)
(17) Lower h-f energy calculated from PTV =   51375939.407257   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   67824976255.875536
(19) H-f difference taken from experiment =   0.110900
(20) H-f difference calculated from PTV   =   0.110927   PTV2 Eq (37)
(21) H-f error magnitude   =   0.000027

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3D5/2
QUANTUM NUMBERS
(1) Quantum Mechanics: n = 3 L = 2 Sp = 0.5   Sommerfeld: n(phi) = 3 n(r) = 0

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers =   5.9164711126492712 PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers =   6.0002951168570048   PTV1 d2-bar Eq (86)

 emult =   0.3745970500

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments   =   365538536.306861
(5) New PTV f-s energy no adjustments   =   365538536.289994   PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass   =   365339566.160507
(7) Relativistic reduced mass multiplier   =   0.9994556786199280   PTV2 (1 + Mr)^(-1) Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   365339565.922954   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier   =   1.00013601   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
    and adjusted velocity   =   365389256.699024   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy   =   365339565.452000   Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
    and average h-f energy   =   49691.247024
(13) Exit potential   =   49691.247025   PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude   =   0.000000

(15) H-f mid-point PTV   =   365339565.452000

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Lower h-f energy from experiment   =   365339564.100300   Horbatsch and Hessels (2016)
(17) Lower h-f energy calculated from PTV =   365339564.099436   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   422715915879.764096
(19) H-f difference taken from experiment =   1.351700
(20) H-f difference calculated from PTV   =   1.352564   PTV2 Eq (37)
(21) H-f error magnitude   =   0.000864

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4D5/2
QUANTUM NUMBERS
(1) Quantum Mechanics: n = 4 L = 2 Sp = 0.5   Sommerfeld: n(phi) = 3 n(r) = 1

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers =   7.9377245304992704   PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers =   8.0003988515982480   PTV1 d2-bar Eq (86)

 emult =   0.3745970500

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments   =   205615521.724133
(5) New PTV f-s energy no adjustments   =   205615521.709497   PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass   =   205503600.965070
(7) Relativistic reduced mass multiplier   =   0.9994556789720584   PTV2 $(1 + Mr)^{-1}$ Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   205503600.898058   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier   =   1.00010201   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
    and adjusted velocity   =   205524564.025718   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy   =   205503600.601850   Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
    and average h-f energy   =   20963.423868
(13) Exit potential   =   20963.423868   PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude   =   0.000000
(15) H-f mid-point PTV   =   205503600.601850

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Lower h-f energy from experiment   =   205503600.031500   Horbatsch and Hessels (2016)
(17) Lower h-f energy calculated from PTV =   205503600.031238   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   784923912693.473536
(19) H-f difference taken from experiment =   0.570350
(20) H-f difference calculated from PTV   =   0.570612   PTV2 Eq (37)
(21) H-f error magnitude   =   0.000262

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5D5/2
QUANTUM NUMBERS
(1) Quantum Mechanics: n = 5 L = 2 Sp = 0.5   Sommerfeld: n(phi) = 3 n(r) = 2

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers =   9.9504353224891328   PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers =   10.00050369579473360   PTV1 d2-bar Eq (86)

 emult =   0.3745970500

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments   =   131593935.343814
(5) New PTV f-s energy no adjustments   =   131593935.354022   PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass   =   131522306.057230
(7) Relativistic reduced mass multiplier   =   0.9994556791350378   PTV2 $(1 + Mr)^{\wedge}(-1)$ Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   131522306.050263   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier   =   1.00008161   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
     and adjusted velocity   =   131533039.117004   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy   =   131522305.871950   Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
     and average h-f energy   =   10733.245054
(13) Exit potential   =   10733.245054   PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude   =   0.000000
(15) H-f mid-point PTV   =   131522305.871950

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Lower h-f energy from experiment   =   131522305.580000   Horbatsch and Hessels (2016)
(17) Lower h-f energy calculated from PTV =   131522305.579797   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   648898141312.649728
(19) H-f difference taken from experiment =   0.291950
(20) H-f difference calculated from PTV   =   0.292153   PTV2 Eq (37)
(21) H-f error magnitude   =   0.000203

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6D5/2
QUANTUM NUMBERS
(1) Quantum Mechanics: n = 6 L = 2 Sp = 0.5   Sommerfeld: n(phi) = 3 n(r) = 3

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers =   11.9589174920513680 PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers   =   12.0006090947191888 PTV1 d2-bar Eq (86)

 emult =   0.3745970500

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments   =   91384667.867097
(5) New PTV f-s energy no adjustments   =   91384667.866803   PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass   =   91334925.312115
(7) Relativistic reduced mass multiplier   =   0.9994556792235676   PTV2 $(1 + Mr)^{\wedge}(-1)$ Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   91334925.304920   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier   =   1.00006801   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
     and adjusted velocity   =   91341136.548634   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy   =   91334925.192250   Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
     and average h-f energy   =   6211.356384
(13) Exit potential   =   6211.356019   PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude   =   0.000365
(15) H-f mid-point PTV   =   91334925.192615

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Lower h-f energy from experiment   =   91334925.023300   Horbatsch and Hessels (2016)
(17) Lower h-f energy calculated from PTV =   91334925.023545   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   372357879948.899648

(19) H-f difference taken from experiment =   0.168950
(20) H-f difference calculated from PTV =   0.169070   PTV2 Eq (37)
(21) H-f error magnitude   =   0.000120

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7D5/2
QUANTUM NUMBERS
(1) Quantum Mechanics: n = 7 L = 2 Sp = 0.5   Sommerfeld: n(phi) = 3 n(r) = 4

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers =   13.9649956729696720   PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers   =   14.0007148106311456   PTV1 d2-bar Eq (86)

 emult =   0.3745970500

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments   =   67139747.364169
(5) New PTV f-s energy no adjustments   =   67139747.369835   PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass   =   67103201.818263
(7) Relativistic reduced mass multiplier   =   0.9994556792769472   PTV2 (1 + Mr)^(-1) Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   67103201.820732   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier   =   1.00005829   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
    and adjusted velocity   =   67107113.264497   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy   =   67103201.745800   Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
    and average h-f energy   =   3911.518697
(13) Exit potential   =   3911.518089   PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude   =   0.000609
(15) H-f mid-point PTV   =   67103201.746409

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Lower h-f energy from experiment   =   67103201.639400   Horbatsch and Hessels (2016)
(17) Lower h-f energy calculated from PTV =   67103201.639939   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   124443021880.318128
(19) H-f difference taken from experiment =   0.106400
(20) H-f difference calculated from PTV   =   0.106469   PTV2 Eq (37)
(21) H-f error magnitude   =   0.000069

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8D5/2
QUANTUM NUMBERS
(1) Quantum Mechanics: n = 8 L = 2 Sp = 0.5   Sommerfeld: n(phi) = 3 n(r) = 5

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers =   15.9695758490998720   PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers   =   16.0008207246602976   PTV1 d2-bar Eq (86)

 emult =   0.3745970500

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments   =   51403862.601317
(5) New PTV f-s energy no adjustments   =   51403862.606216   PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass   =   51375882.421256
(7) Relativistic reduced mass multiplier   =   0.9994556793115924   PTV2 (1 + Mr)^(-1) Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   51375882.424514   PTV2 (D = 1) Eq (13)

(9) Velocity D multiplier   =   1.000051   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
    and adjusted velocity   =   51378502.781152   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy   =   51375882.372400   Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
    and average h-f energy   =   2620.408752
(13) Exit potential   =   2620.407994   PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude   =   0.000758
(15) H-f mid-point PTV   =   51375882.373158

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Lower h-f energy from experiment   =   51375882.301100   Horbatsch and Hessels (2016)
(17) Lower h-f energy calculated from PTV =   51375882.301832   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   70162334111.203232
(19) H-f difference taken from experiment =   0.071300
(20) H-f difference calculated from PTV   =   0.071326   PTV2 Eq (37)
(21) H-f error magnitude   =   0.000026

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NEGATIVE GAMMA (emult)

1S1/2
QUANTUM NUMBERS
(1) Quantum Mechanics: n = 1 L = 0 Sp = 0.5   Sommerfeld: n(phi) = 1 n(r) = 0

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers =   1.7324035537303600   PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers =   2.0000332836086264   PTV1 d2-bar Eq (86)

 emult =   -0.5490120000

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments   =   3289885758.535002
(5) New PTV f-s energy no adjustments   =   3289885758.260552   PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass   =   3288095006.026450
(7) Relativistic reduced mass multiplier   =   0.9994556721864408   PTV2 (1 + Mr)^(-1) Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   3288094955.765224   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier   =   0.99940209   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
    and adjusted velocity   =   3286128960.524324   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy   =   3288087212.213100 Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
    and average h-f energy   =   1958251.688776
(13) Exit potential   =   1958251.688776   PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude   =   0.000000
(15) H-f mid-point PTV   =   3288087212.213100

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Higher h-f energy from experiment   =   3288087922.416000   Horbatsch and Hessels (2016)
(17) Higher h-f energy calculated from PTV =   3288087922.232933   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   17961132850.081810
(19) H-f difference taken from experiment =   710.202900
(20) H-f difference calculated from PTV   =   710.019833   PTV2 Eq (37)
(21) H-f error magnitude   =   0.183066

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2S1/2
QUANTUM NUMBERS
(1) Quantum Mechanics: n = 2 L = 0 Sp = 0.5   Sommerfeld: n(phi) = 1 n(r) = 1

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers =   3.8732511083690200   PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers =   4.0001231494850968   PTV1 d2-bar Eq (86)

 emult =   -0.5490120000

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments   =   822474177.055886
(5) New PTV f-s energy no adjustments   =   822474177.045523   PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass   =   822026487.438713
(7) Relativistic reduced mass multiplier   =   0.9994556776146438   PTV2 (1 + Mr)^(-1) Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   822026481.849575   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier   =   0.99970102   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
     and adjusted velocity   =   821780713.251798   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy   =   822025488.313800   Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
     and average h-f energy   =   244775.062003
(13) Exit potential   =   244775.062003   PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude   =   0.000000
(15) H-f mid-point PTV   =   822025488.313800

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Higher h-f energy from experiment   =   822025577.092200   Horbatsch and Hessels (2016)
(17) Higher h-f energy calculated from PTV   =   822025577.061922   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   27149405415.983552
(19) H-f difference taken from experiment =   88.778400
(20) H-f difference calculated from PTV   =   88.748122   PTV2 Eq (37)
(21) H-f error magnitude   =   0.030278

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3S1/2
QUANTUM NUMBERS
(1) Quantum Mechanics: n = 3 L = 0 Sp = 0.5   Sommerfeld: n(phi) = 1 n(r) = 2

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers =   5.9163991001174104   PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers =   6.0002241103507824   PTV1 d2-bar Eq (86)

 emult =   -0.5490120000

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments   =   365542862.051824
(5) New PTV f-s energy no adjustments   =   365542862.054546   PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass   =   365343889.550878
(7) Relativistic reduced mass multiplier   =   0.9994556786201884   PTV2 (1 + Mr)^(-1) Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   365343888.289990   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier   =   0.99980068   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
     and adjusted velocity   =   365271066.392270   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)

(11) Average h-f energy   =   365343591.599600   Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
      and average h-f energy   =   72525.207330
(13) Exit potential   =   72525.207330   PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude =   0.000000
(15) H-f mid-point PTV   =   365343591.599600

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Higher h-f energy from experiment   =   365343617.904300   Horbatsch and Hessels (2016)
(17) Higher h-f energy calculated from PTV =   365343617.894686   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   38001009267.388160
(19) H-f difference taken from experiment =   26.304700
(20) H-f difference calculated from PTV   =   26.295086   PTV2 Eq (37)
(21) H-f error magnitude   =   0.009614

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4S1/2
QUANTUM NUMBERS
(1) Quantum Mechanics: n = 4 L = 0 Sp = 0.5   Sommerfeld: n(phi) = 1 n(r) = 3

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers =   7.9376529632206608   PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers   =   8.0003278449755088   PTV1 d2-bar Eq (86)

 emult =   -0.5490120000

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments   =   205617346.644788
(5) New PTV f-s energy no adjustments   =   205617346.640405   PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass   =   205505424.892383
(7) Relativistic reduced mass multiplier   =   0.9994556789721682   PTV2 $(1 + Mr)^{-1}$ Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   205505424.462619   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier   =   0.9998505   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
      and adjusted velocity   =   205474702.418402   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy   =   205505298.855200   Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
      and average h-f energy   =   30596.436798
(13) Exit potential   =   30596.445663   PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude =   0.008865
(15) H-f mid-point PTV   =   205505298.864065

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Higher h-f energy from experiment   =   205505309.952500   Horbatsch and Hessels (2016)
(17) Higher h-f energy calculated from PTV =   205505309.957148   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   44210872203.422968
(19) H-f difference taken from experiment =   11.097300
(20) H-f difference calculated from PTV   =   11.093084   PTV2 Eq (37)
(21) H-f error magnitude   =   0.004216

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5S1/2
QUANTUM NUMBERS
(1) Quantum Mechanics: n = 5 L = 0 Sp = 0.5   Sommerfeld: n(phi) = 1 n(r) = 4

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)

(2) Horizontal distance between p-e Sp-2 centers =   9.9503639585056512 PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers =   10.0004326891036560 PTV1 d2-bar Eq (86)

 emult =   -0.5490120000

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments =   131594869.718334
(5) New PTV f-s energy no adjustments =   131594869.717838   PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass =   131523239.923151
(7) Relativistic reduced mass multiplier =   0.9994556791350940   PTV2 $(1 + Mr)^{-1}$ Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   131523239.742134   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier =   0.9998804   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
    and adjusted velocity =   131507509.965646   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy =   131523175.306400   Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
    and average h-f energy =   15665.340754
(13) Exit potential =   15665.354811   PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude =   0.014058
(15) H-f mid-point PTV =   131523175.320458

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Higher h-f energy from experiment =   131523180.988200   Horbatsch and Hessels (2016)
(17) Higher h-f energy calculated from PTV =   131523181.000066   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   11084061606.235370
(19) H-f difference taken from experiment =   5.681800
(20) H-f difference calculated from PTV =   5.679608   PTV2 Eq (37)
(21) H-f error magnitude =   0.002192

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6S1/2
QUANTUM NUMBERS
(1) Quantum Mechanics: n = 6 L = 0 Sp = 0.5   Sommerfeld: n(phi) = 1 n(r) = 5

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers =   11.9588462377676944 PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers =   12.0005380879831920 PTV1 d2-bar Eq (86)

 emult =   -0.5490120000

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments =   91385208.595505
(5) New PTV f-s energy no adjustments =   91385208.586172   PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass =   91335465.746194
(7) Relativistic reduced mass multiplier =   0.9994556792236000   PTV2 $(1 + Mr)^{-1}$ Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   91335465.647789   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier =   0.99990034   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
    and adjusted velocity =   91326362.734225   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy =   91335428.313650   Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
    and average h-f energy =   9065.579425
(13) Exit potential =   9065.594956   PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude =   0.015531
(15) H-f mid-point PTV =   91335428.329181

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Higher h-f energy from experiment   =   91335431.601700   Horbatsch and Hessels (2016)
(17) Higher h-f energy calculated from PTV =   91335431.615971   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   6400143111.349888
(19) H-f difference taken from experiment =   3.288050
(20) H-f difference calculated from PTV   =   3.286790   PTV2 Eq (37)
(21) H-f error magnitude   =   0.001260

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2P1/2
QUANTUM NUMBERS
(1) Quantum Mechanics: n = 2 L = 1 Sp = -0.5   Sommerfeld: n(phi) = 1 n(r) = 1

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers =   3.8732511083690200   PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers   =   4.0001231494850968   PTV1 d2-bar Eq (86)

 emult =   -0.2048164400

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments   =   822474177.055886
(5) New PTV f-s energy no adjustments   =   822474177.045523   PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass   =   822026487.438713
(7) Relativistic reduced mass multiplier   =   0.9994556776143044   PTV2 (1 + Mr)^(-1) Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   822026484.413397   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier   =   0.99988846   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
    and adjusted velocity   =   821934792.777153   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy   =   822026516.551150   Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
    and average h-f energy   =   91723.773997
(13) Exit potential   =   91723.773996   PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude   =   0.000001
(15) H-f mid-point PTV   =   822026516.551149

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Higher h-f energy from experiment   =   822026546.135900   Horbatsch and Hessels (2016)
(17) Higher h-f energy calculated from PTV =   822026546.133857   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   402313795865.109696
(19) H-f difference taken from experiment =   29.584750
(20) H-f difference calculated from PTV   =   29.582707   PTV2 Eq (37)
(21) H-f error magnitude   =   0.002043

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3P1/2
QUANTUM NUMBERS
(1) Quantum Mechanics: n = 3 L = 1 Sp = -0.5   Sommerfeld: n(phi) = 1 n(r) = 2

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers =   5.9163991001174104   PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers   =   6.0002241103507824   PTV1 d2-bar Eq (86)

 emult =   -0.2048164400

FINE STRUCTURE (ALL ENERGIES IN MHz)

(4) Sommerfeld f-s energy no adjustments   =   365542862.051824
(5) New PTV f-s energy no adjustments   =   365542862.054546   PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass   =   365343889.550878
(7) Relativistic reduced mass multiplier   =   0.9994556786200880   PTV2 (1 + Mr)^(-1) Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   365343888.897758   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier   =   0.99992564   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
    and adjusted velocity   =   365316720.843363   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy   =   365343897.705100   Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
    and average h-f energy   =   27176.861737
(13) Exit potential   =   27176.861737   PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude   =   0.000000
(15) H-f mid-point PTV   =   365343897.705100

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Higher h-f energy from experiment   =   365343906.471000   Horbatsch and Hessels (2016)
(17) Higher h-f energy calculated from PTV =   365343906.470128   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   419165262473.347776
(19) H-f difference taken from experiment =   8.765900
(20) H-f difference calculated from PTV   =   8.765029   PTV2 Eq (37)
(21) H-f error magnitude   =   0.000871

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4P1/2
QUANTUM NUMBERS
(1) Quantum Mechanics: n = 4 L = 1 Sp = -0.5   Sommerfeld: n(phi) = 1 n(r) = 3

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers   =   7.9376529632206608   PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers   =   8.0003278449755088   PTV1 d2-bar Eq (86)

 emult =   -0.2048164400

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments   =   205617346.644788
(5) New PTV f-s energy no adjustments   =   205617346.640405   PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass   =   205505424.892383
(7) Relativistic reduced mass multiplier   =   0.9994556789721258   PTV2 (1 + Mr)^(-1) Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   205505424.670952   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier   =   0.99994423   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
    and adjusted velocity   =   205493963.123335   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy   =   205505428.231800   Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
    and average h-f energy   =   11465.108465
(13) Exit potential   =   11465.108465   PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude   =   0.000000
(15) H-f mid-point PTV   =   205505428.231800

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Higher h-f energy from experiment   =   205505431.929900   Horbatsch and Hessels (2016)
(17) Higher h-f energy calculated from PTV =   205505431.929495   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   506807146944.176000
(19) H-f difference taken from experiment =   3.698100

(20) H-f difference calculated from PTV   =   3.697695   PTV2 Eq (37)
(21) H-f error magnitude   =   0.000405

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5P1/2
QUANTUM NUMBERS
(1) Quantum Mechanics: n = 5 L = 1 Sp = -0.5   Sommerfeld: n(phi) = 1 n(r) = 4

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers =   9.9503639585056512 PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers =   10.0004326891036560 PTV1 d2-bar Eq (86)

 emult =   -0.2048164400

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments   =   131594869.718334
(5) New PTV f-s energy no adjustments   =   131594869.717838   PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass   =   131523239.923151
(7) Relativistic reduced mass multiplier   =   0.9994556791350722   PTV2 $(1 + Mr)^{-1}$ Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   131523239.831407   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier   =   0.99995538   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
    and adjusted velocity   =   131517371.513011   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy   =   131523241.607050   Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
    and average h-f energy   =   5870.094039
(13) Exit potential   =   5870.093147   PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude   =   0.000892
(15) H-f mid-point PTV   =   131523241.606158

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Higher h-f energy from experiment   =   131523243.500500   Horbatsch and Hessels (2016)
(17) Higher h-f energy calculated from PTV =   131523243.499361   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   115428758675.933600
(19) H-f difference taken from experiment =   1.893450
(20) H-f difference calculated from PTV   =   1.893203   PTV2 Eq (37)
(21) H-f error magnitude   =   0.000247

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6P1/2
QUANTUM NUMBERS
(1) Quantum Mechanics: n = 6 L = 1 Sp = -0.5   Sommerfeld: n(phi) = 1 n(r) = 5

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers =   11.9588462377676944 PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers =   12.0005380879831920 PTV1 d2-bar Eq (86)

 emult =   -0.2048164400

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments   =   91385208.595505
(5) New PTV f-s energy no adjustments   =   91385208.586172   PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass   =   91335465.746194
(7) Relativistic reduced mass multiplier   =   0.9994556792235876   PTV2 $(1 + Mr)^{-1}$ Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   91335465.692108   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier   =   0.99996282   PTV2 Eq (13)

(10) New PTV f-s energy with relativistic reduced mass
and adjusted velocity   =   91332069.672574   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy   =   91335466.701500   Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
and average h-f energy   =   3397.028926
(13) Exit potential   =   3397.027639   PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude   =   0.001287
(15) H-f mid-point PTV   =   91335466.700213

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Higher h-f energy from experiment   =   91335467.797200   Horbatsch and Hessels (2016)
(17) Higher h-f energy calculated from PTV =   91335467.795810   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   65690885858.281528
(19) H-f difference taken from experiment =   1.095700
(20) H-f difference calculated from PTV   =   1.095597   PTV2 Eq (37)
(21) H-f error magnitude   =   0.000103

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7P1/2
QUANTUM NUMBERS
(1) Quantum Mechanics: n = 7 L = 1 Sp = -0.5   Sommerfeld: n(phi) = 1 n(r) = 6

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers =   13.9649244845825792 PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers =   14.0006438038634048 PTV1 d2-bar Eq (86)

 emult =   -0.2048164400

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments   =   67140087.867507
(5) New PTV f-s energy no adjustments   =   67140087.880927   PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass   =   67103542.136257
(7) Relativistic reduced mass multiplier   =   0.9994556792769598   PTV2 (1 + Mr)^(-1) Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   67103542.125220   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier   =   0.99996813   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
and adjusted velocity   =   67101403.523239   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy   =   67103542.752550   Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
and average h-f energy   =   2139.229311
(13) Exit potential   =   2139.228187   PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude   =   0.001123
(15) H-f mid-point PTV   =   67103542.751427

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Higher h-f energy from experiment   =   67103543.442300   Horbatsch and Hessels (2016)
(17) Higher h-f energy calculated from PTV =   67103543.441362   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   71530566364.930816
(19) H-f difference taken from experiment =   0.689750
(20) H-f difference calculated from PTV   =   0.689935   PTV2 Eq (37)
(21) H-f error magnitude   =   0.000185

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2P3/2

QUANTUM NUMBERS
(1) Quantum Mechanics: n = 2 L = 1 Sp = 0.5   Sommerfeld: n(phi) = 2 n(r) = 0

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers =   3.8733061076133600 PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers =   4.0001764043401488 PTV1 d2-bar Eq (86)

 emult =   -0.2336101424

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments   =   822463227.394777
(5) New PTV f-s energy no adjustments   =   822463227.397312   PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass   =   822015543.737730
(7) Relativistic reduced mass multiplier   =   0.9994556776143570   PTV2 $(1 + Mr)^{-1}$ Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   822015541.903204   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier   =   0.99987278   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
    and adjusted velocity   =   821910961.787048   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy   =   822015535.670900   Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
    and average h-f energy   =   104573.883853
(13) Exit potential   =   104573.883853   PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude   =   0.000000
(15) H-f mid-point PTV   =   822015535.670900

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Higher h-f energy from experiment   =   822015547.496700   Horbatsch and Hessels (2016)
(17) Higher h-f energy calculated from PTV =   822015547.503353   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   123556526686.649088
(19) H-f difference taken from experiment =   11.825800
(20) H-f difference calculated from PTV   =   11.832453   PTV2 Eq (37)
(21) H-f error magnitude   =   0.006653

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3P3/2
QUANTUM NUMBERS
(1) Quantum Mechanics: n = 3 L = 1 Sp = 0.5   Sommerfeld: n(phi) = 2 n(r) = 1

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers =   5.9164531096983408 PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers =   6.0002773654076936 PTV1 d2-bar Eq (86)

 emult =   -0.2336101424

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments   =   365539617.722525
(5) New PTV f-s energy no adjustments   =   365539617.710736   PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass   =   365340646.987534
(7) Relativistic reduced mass multiplier   =   0.9994556786201034   PTV2 $(1 + Mr)^{-1}$ Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   365340646.544107   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier   =   0.99991518   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
    and adjusted velocity   =   365309659.480071   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy   =   365340644.107950   Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections

and average h-f energy   =   30984.627879
(13) Exit potential   =   30984.627879   PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude   =   0.000000
(15) H-f mid-point PTV   =   365340644.107950

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Higher h-f energy from experiment   =   365340647.611900   Horbatsch and Hessels (2016)
(17) Higher h-f energy calculated from PTV =   365340647.613837   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   188614301583.676352
(19) H-f difference taken from experiment =   3.503950
(20) H-f difference calculated from PTV   =   3.505887   PTV2 Eq (37)
(21) H-f error magnitude   =   0.001937

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4P3/2
QUANTUM NUMBERS
(1) Quantum Mechanics: n = 4 L = 1 Sp = 0.5   Sommerfeld: n(phi) = 2 n(r) = 2

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers =   7.9377066388691312   PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers =   8.0003811001296544   PTV1 d2-bar Eq (86)

 emult =   -0.2336101424

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments   =   205615977.950867
(5) New PTV f-s energy no adjustments   =   205615977.934378   PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass   =   205504056.943471
(7) Relativistic reduced mass multiplier   =   0.9994556789721326   PTV2 (1 + Mr)^(-1) Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   205504056.779536   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier   =   0.99993639   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
    and adjusted velocity   =   205490984.046214   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy   =   205504055.622150   Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
    and average h-f energy   =   13071.575936
(13) Exit potential   =   13071.575936   PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude   =   0.000000
(15) H-f mid-point PTV   =   205504055.622150

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Higher h-f energy from experiment   =   205504057.100400   Horbatsch and Hessels (2016)
(17) Higher h-f energy calculated from PTV =   205504057.101188   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   260643026523.264640
(19) H-f difference taken from experiment =   1.478250
(20) H-f difference calculated from PTV   =   1.479038   PTV2 Eq (37)
(21) H-f error magnitude   =   0.000788

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5P3/2
QUANTUM NUMBERS
(1) Quantum Mechanics: n = 5 L = 1 Sp = 0.5   Sommerfeld: n(phi) = 2 n(r) = 3

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers =   9.9504174816877120   PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers =   10.0004859443149632   PTV1 d2-bar Eq (86)

emult =   -0.2336101424

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments   =   131594168.930585
(5) New PTV f-s energy no adjustments   =   131594168.941192   PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass   =   131522539.516855
(7) Relativistic reduced mass multiplier   =   0.9994556791350758   PTV2 $(1 + Mr)^{\wedge}(-1)$ Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   131522539.464385   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier   =   0.99994911   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
    and adjusted velocity   =   131515846.206529   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy   =   131522538.831750   Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
    and average h-f energy   =   6692.625221
(13) Exit potential   =   6692.624227   PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude   =   0.000994
(15) H-f mid-point PTV   =   131522538.830756

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Higher h-f energy from experiment   =   131522539.588600   Horbatsch and Hessels (2016)
(17) Higher h-f energy calculated from PTV =   131522539.588021   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   227119762806.494080
(19) H-f difference taken from experiment =   0.756850
(20) H-f difference calculated from PTV   =   0.757265   PTV2 Eq (37)
(21) H-f error magnitude   =   0.000415

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6P3/2
QUANTUM NUMBERS
(1) Quantum Mechanics: n = 6 L = 1 Sp = 0.5   Sommerfeld: n(phi) = 2 n(r) = 4

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers =   11.9588996786783616 PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers =   12.0005913432321440 PTV1 d2-bar Eq (86)

 emult =   -0.2336101424

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments   =   91384803.042340
(5) New PTV f-s energy no adjustments   =   91384803.044546   PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass   =   91335060.413780
(7) Relativistic reduced mass multiplier   =   0.9994556792235896   PTV2 $(1 + Mr)^{\wedge}(-1)$ Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   91335060.384709   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier   =   0.99995759   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
    and adjusted velocity   =   91331186.964930   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy   =   91335060.003300   Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
    and average h-f energy   =   3873.038370
(13) Exit potential   =   3873.036852   PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude   =   0.001518
(15) H-f mid-point PTV   =   91335060.001782

HYPERFINE SHIFT (ALL ENERGIES IN MHz)

(16) Higher h-f energy from experiment   =   91335060.441300   Horbatsch and Hessels (2016)
(17) Higher h-f energy calculated from PTV =   91335060.440013   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   70984761078.276064
(19) H-f difference taken from experiment =   0.438000
(20) H-f difference calculated from PTV   =   0.438231   PTV2 Eq (37)
(21) H-f error magnitude   =   0.000231

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7P3/2
QUANTUM NUMBERS
(1) Quantum Mechanics: n = 7 L = 2 Sp = -0.5   Sommerfeld: n(phi) = 2 n(r) = 5

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers =   13.9649778760733312 PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers   =   14.0006970591389616 PTV1 d2-bar Eq (86)

 emult =   -0.2336101424

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments   =   67139832.496862
(5) New PTV f-s energy no adjustments   =   67139832.496326   PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass   =   67103286.904616
(7) Relativistic reduced mass multiplier   =   0.9994556792769612   PTV2 (1 + Mr)^(-1) Eq (13)
(8) New PTV f-s energy relativ. reduced mass   =   67103286.886867   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier   =   0.99996365   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
    and adjusted velocity   =   67100847.645890   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy   =   67103286.639850 Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
    and average h-f energy   =   2438.993960
(13) Exit potential   =   2438.992361 PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude   =   0.001600
(15) H-f mid-point PTV   =   67103286.638250

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Higher h-f energy from experiment   =   67103286.915700   Horbatsch and Hessels (2016)
(17) Higher h-f energy calculated from PTV =   67103286.914220   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   45348559760.113704
(19) H-f difference taken from experiment =   0.275850
(20) H-f difference calculated from PTV   =   0.275970   PTV2 Eq (37)
(21) H-f error magnitude   =   0.000120

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3D3/2
QUANTUM NUMBERS
(1) Quantum Mechanics: n = 3 L = 2 Sp = -0.5   Sommerfeld: n(phi) = 2 n(r) = 1

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers =   5.9164531096983408 PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers   =   6.0002773654076936 PTV1 d2-bar Eq (86)

 emult =   -0.1836735000

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments   =   365539617.722525
(5) New PTV f-s energy no adjustments   =   365539617.710736   PTV1 Eq (54)

(6) Sommerfeld f-s energy reduced mass   =   365340646.987534
(7) Relativistic reduced mass multiplier   =   0.9994556786200890   PTV2 $(1 + Mr)^{-1}$ Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   365340646.573495   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier   =   0.99993331   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
    and adjusted velocity   =   365316283.212186   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy   =   365340649.089800   Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
    and average h-f energy   =   24365.877614
(13) Exit potential   =   24365.877614   PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude   =   0.000000
(15) H-f mid-point PTV   =   365340649.089800

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Higher h-f energy from experiment   =   365340651.193100   Horbatsch and Hessels (2016)
(17) Higher h-f energy calculated from PTV =   365340651.193332   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   1574466739955.637248
(19) H-f difference taken from experiment =   2.103300
(20) H-f difference calculated from PTV =   2.103532   PTV2 Eq (37)
(21) H-f error magnitude   =   0.000232

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4D3/2
QUANTUM NUMBERS
(1) Quantum Mechanics: n = 4 L = 2 Sp = -0.5   Sommerfeld: n(phi) = 2 n(r) = 2

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers =   7.9377066388691312 PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers =   8.0003811001296544 PTV1 d2-bar Eq (86)

 emult =   -0.1836735000

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments   =   205615977.950867
(5) New PTV f-s energy no adjustments   =   205615977.934378   PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass   =   205504056.943471
(7) Relativistic reduced mass multiplier   =   0.9994556789721264   PTV2 $(1 + Mr)^{-1}$ Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   205504056.791161   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier   =   0.99994998   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
    and adjusted velocity   =   205493778.458114   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy   =   205504057.762150   Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
    and average h-f energy   =   10279.304036
(13) Exit potential   =   10279.304036   PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude   =   0.000000
(15) H-f mid-point PTV   =   205504057.762150

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Higher h-f energy from experiment   =   205504058.649500   Horbatsch and Hessels (2016)
(17) Higher h-f energy calculated from PTV =   205504058.649573   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   2816818611796.838400
(19) H-f difference taken from experiment =   0.887350
(20) H-f difference calculated from PTV =   0.887423   PTV2 Eq (37)
(21) H-f error magnitude   =   0.000073

5D3/2
QUANTUM NUMBERS
(1) Quantum Mechanics: n = 5 L = 2 Sp = -0.5   Sommerfeld: n(phi) = 2 n(r) = 3

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers =   9.9504174816877120 PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers   =   10.0004859443149632 PTV1 d2-bar Eq (86)

 emult =   -0.1836735000

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments   =   131594168.930585
(5) New PTV f-s energy no adjustments   =   131594168.941192   PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass   =   131522539.516855
(7) Relativistic reduced mass multiplier   =   0.9994556791350726   PTV2 $(1 + Mr)^{-1}$ Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   131522539.469718   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier   =   0.99995999   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
     and adjusted velocity   =   131517276.952313   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy   =   131522539.938050 Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
     and average h-f energy   =   5262.985737
(13) Exit potential   =   5262.985737   PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude   =   0.000000
(15) H-f mid-point PTV   =   131522539.938050

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Higher h-f energy from experiment   =   131522540.392400   Horbatsch and Hessels (2016)
(17) Higher h-f energy calculated from PTV =   131522540.392409   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   14710547126880.131072
(19) H-f difference taken from experiment   =   0.454350
(20) H-f difference calculated from PTV   =   0.454359   PTV2 Eq (37)
(21) H-f error magnitude   =   0.000009

6D3/2
QUANTUM NUMBERS
(1) Quantum Mechanics: n = 6 L = 2 Sp = -0.5   Sommerfeld: n(phi) = 2 n(r) = 4

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers =   11.9588996786783616 PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers   =   12.0005913432321440 PTV1 d2-bar Eq (86)

 emult =   -0.1836735000

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments   =   91384803.042340
(5) New PTV f-s energy no adjustments   =   91384803.044546   PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass   =   91335060.413780
(7) Relativistic reduced mass multiplier   =   0.9994556792235876   PTV2 $(1 + Mr)^{-1}$ Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   91335060.387464   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier   =   0.99996666   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
     and adjusted velocity   =   91332014.945290   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy   =   91335060.647200   Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
    and average h-f energy   =   3045.701910
(13) Exit potential   =   3045.701753 PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude   =   0.000157
(15) H-f mid-point PTV   =   91335060.647043

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Higher h-f energy from experiment   =   91335060.910100   Horbatsch and Hessels (2016)
(17) Higher h-f energy calculated from PTV =   91335060.909982   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   771672186963.063936
(19) H-f difference taken from experiment =   0.262900
(20) H-f difference calculated from PTV   =   0.262939   PTV2 Eq (37)
(21) H-f error magnitude   =   0.000039

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7D3/2
QUANTUM NUMBERS
(1) Quantum Mechanics: n = 7 L = 2 Sp = -0.5   Sommerfeld: n(phi) = 2 n(r) = 5

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers =   13.9649778760733312 PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers   =   14.0006970591389616 PTV1 d2-bar Eq (86)

 emult =   -0.1836735000

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments   =   67139832.496862
(5) New PTV f-s energy no adjustments   =   67139832.496326   PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass   =   67103286.904616
(7) Relativistic reduced mass multiplier   =   0.9994556792769598   PTV2 (1 + Mr)^(-1) Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   67103286.888425   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier   =   0.99997142   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
    and adjusted velocity   =   67101369.057318   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy   =   67103287.046850   Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
    and average h-f energy   =   1917.989532
(13) Exit potential   =   1917.989289 PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude   =   0.000243
(15) H-f mid-point PTV   =   67103287.046607

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Higher h-f energy from experiment   =   67103287.212400   Horbatsch and Hessels (2016)
(17) Higher h-f energy calculated from PTV =   67103287.212189   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   317710270600.387392
(19) H-f difference taken from experiment =   0.165550
(20) H-f difference calculated from PTV   =   0.165582   PTV2 Eq (37)
(21) H-f error magnitude   =   0.000032

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8D3/2
QUANTUM NUMBERS

(1) Quantum Mechanics: n = 8 L = 2 Sp = -0.5   Sommerfeld: n(phi) = 2 n(r) = 6

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers =   15.9695580628726016 PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers =   16.0008029731643200 PTV1 d2-bar Eq (86)

 emult =   -0.1836735000

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments =   51403919.639947
(5) New PTV f-s energy no adjustments =   51403919.634302   PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass =   51375939.428839
(7) Relativistic reduced mass multiplier =   0.9994556793116008   PTV2 $(1 + Mr)^{-1}$ Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   51375939.413906   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier =   0.99997499   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
    and adjusted velocity =   51374654.616113   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy =   51375939.517400 Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
    and average h-f energy =   1284.901287
(13) Exit potential =   1284.901042 PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude =   0.000245
(15) H-f mid-point PTV =   51375939.517155

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Higher h-f energy from experiment =   51375939.628300   Horbatsch and Hessels (2016)
(17) Higher h-f energy calculated from PTV =   51375939.628082   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   235399648065.257600
(19) H-f difference taken from experiment =   0.110900
(20) H-f difference calculated from PTV =   0.110927   PTV2 Eq (37)
(21) H-f error magnitude =   0.000027

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3D5/2
QUANTUM NUMBERS
(1) Quantum Mechanics: n = 3 L = 2 Sp = 0.5   Sommerfeld: n(phi) = 3 n(r) = 0

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers =   5.9164711126492712 PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers =   6.0002951168570048 PTV1 d2-bar Eq (86)

 emult =   -0.1950811000

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments =   365538536.306861
(5) New PTV f-s energy no adjustments =   365538536.289994   PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass =   365339566.160507
(7) Relativistic reduced mass multiplier =   0.9994556786200946   PTV2 $(1 + Mr)^{-1}$ Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   365339565.811235   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier =   0.99992917   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
    and adjusted velocity =   365313689.392794   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy =   365339565.452000 Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
    and average h-f energy =   25876.059206

(13) Exit potential   =   25876.059206   PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude   =   0.000000
(15) H-f mid-point PTV   =   365339565.452000

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Higher h-f energy from experiment   =   365339566.803700   Horbatsch and Hessels (2016)
(17) Higher h-f energy calculated from PTV =   365339566.804564   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   422803395572.332480
(19) H-f difference taken from experiment =   1.351700
(20) H-f difference calculated from PTV   =   1.352564   PTV2 Eq (37)
(21) H-f error magnitude   =   0.000864

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4D5/2
QUANTUM NUMBERS
(1) Quantum Mechanics: n = 4 L = 2 Sp = 0.5   Sommerfeld: n(phi) = 3 n(r) = 1

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers =   7.9377245304992704   PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers =   8.0003988515982480   PTV1 d2-bar Eq (86)

 emult =   -0.1950811000

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments   =   205615521.724133
(5) New PTV f-s energy no adjustments   =   205615521.709497   PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass   =   205503600.965070
(7) Relativistic reduced mass multiplier   =   0.9994556789721288   PTV2 (1 + Mr)^(-1) Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   205503600.836179   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier   =   0.99994688   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
     and adjusted velocity   =   205492684.168772   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy   =   205503600.601850   Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
     and average h-f energy   =   10916.433078
(13) Exit potential   =   10916.433078   PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude   =   0.000000
(15) H-f mid-point PTV   =   205503600.601850

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Higher h-f energy from experiment   =   205503601.172200   Horbatsch and Hessels (2016)
(17) Higher h-f energy calculated from PTV =   205503601.172462   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   785013275419.820672
(19) H-f difference taken from experiment =   0.570350
(20) H-f difference calculated from PTV   =   0.570612   PTV2 Eq (37)
(21) H-f error magnitude   =   0.000262

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5D5/2
QUANTUM NUMBERS
(1) Quantum Mechanics: n = 5 L = 2 Sp = 0.5   Sommerfeld: n(phi) = 3 n(r) = 2

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers =   9.9504353224891328   PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers =   10.00050369579473600   PTV1 d2-bar Eq (86)

emult =   -0.1950811000

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments   =   131593935.343814
(5) New PTV f-s energy no adjustments   =   131593935.354022   PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass   =   131522306.057230
(7) Relativistic reduced mass multiplier   =   0.9994556791350738   PTV2 $(1 + Mr)^{-1}$ Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   131522306.018397   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier   =   0.9999575   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
    and adjusted velocity   =   131516716.669776   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy   =   131522305.871950   Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
    and average h-f energy   =   5589.202174
(13) Exit potential   =   5589.202175   PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude   =   0.000000
(15) H-f mid-point PTV   =   131522305.871950

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Higher h-f energy from experiment   =   131522306.163900   Horbatsch and Hessels (2016)
(17) Higher h-f energy calculated from PTV =   131522306.164103   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   648898144193.466240
(19) H-f difference taken from experiment =   0.291950
(20) H-f difference calculated from PTV   =   0.292153   PTV2 Eq (37)
(21) H-f error magnitude   =   0.000203

6D5/2
QUANTUM NUMBERS
(1) Quantum Mechanics: n = 6 L = 2 Sp = 0.5   Sommerfeld: n(phi) = 3 n(r) = 3

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers =   11.9589174920513680   PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers   =   12.0006090947191888   PTV1 d2-bar Eq (86)

 emult =   -0.1950811000

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments   =   91384667.867097
(5) New PTV f-s energy no adjustments   =   91384667.866803   PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass   =   91334925.312115
(7) Relativistic reduced mass multiplier   =   0.9994556792235884   PTV2 $(1 + Mr)^{-1}$ Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   91334925.287457   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier   =   0.99996459   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
    and adjusted velocity   =   91331690.705315   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy   =   91334925.192250   Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
    and average h-f energy   =   3234.486935
(13) Exit potential   =   3234.486833   PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude   =   0.000102
(15) H-f mid-point PTV   =   91334925.192148

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Higher h-f energy from experiment   =   91334925.361200   Horbatsch and Hessels (2016)

(17) Higher h-f energy calculated from PTV = 91334925.361217 PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in 5243270388806.605824
(19) H-f difference taken from experiment = 0.168950
(20) H-f difference calculated from PTV = 0.169070 PTV2 Eq (37)
(21) H-f error magnitude = 0.000120

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7D5/2
QUANTUM NUMBERS
(1) Quantum Mechanics: n = 7 L = 2 Sp = 0.5 Sommerfeld: n(phi) = 3 n(r) = 4

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers = 13.9649956729696720 PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers = 14.0007148106311456 PTV1 d2-bar Eq (86)

 emult = -0.1950811000

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments = 67139747.364169
(5) New PTV f-s energy no adjustments = 67139747.369835 PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass = 67103201.818263
(7) Relativistic reduced mass multiplier = 0.9994556792769602 PTV2 $(1 + Mr)^{-1}$ Eq (13)
(8) New PTV f-s energy relativ. reduced mass = 67103201.810497 PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier = 0.99996964 PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
    and adjusted velocity = 67101164.870157 PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy = 67103201.745800 Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
    and average h-f energy = 2036.875643
(13) Exit potential = 2036.875428 PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude = 0.000216
(15) H-f mid-point PTV = 67103201.745584

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Higher h-f energy from experiment = 67103201.852200 Horbatsch and Hessels (2016)
(17) Higher h-f energy calculated from PTV = 67103201.852054 PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in 458529645358.297344
(19) H-f difference taken from experiment = 0.106400
(20) H-f difference calculated from PTV = 0.106469 PTV2 Eq (37)
(21) H-f error magnitude = 0.000069

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8D5/2
QUANTUM NUMBERS
(1) Quantum Mechanics: n = 8 L = 2 Sp = 0.5 Sommerfeld: n(phi) = 3 n(r) = 5

BOUND STATE DISTANCES (as a muliplier of a fixed radius for all states)
(2) Horizontal distance between p-e Sp-2 centers = 15.9695758490998720 PTV1 x-bar Eqs (84)/(86)
(3) Shortest distance between p-e Sp-2 centers = 16.0008207246602976 PTV1 d2-bar Eq (86)

 emult = -0.1950811000

FINE STRUCTURE (ALL ENERGIES IN MHz)
(4) Sommerfeld f-s energy no adjustments = 51403862.601317
(5) New PTV f-s energy no adjustments = 51403862.606216 PTV1 Eq (54)
(6) Sommerfeld f-s energy reduced mass = 51375882.421256

(7) Relativistic reduced mass multiplier   =   0.9994556793116010   PTV2 $(1 + Mr)^{\wedge}(-1)$ Eq (13)
(8) New PTV f-s energy relativ. reduced mass =   51375882.418159   PTV2 (D = 1) Eq (13)
(9) Velocity D multiplier   =   0.99997344   PTV2 Eq (13)
(10) New PTV f-s energy with relativistic reduced mass
    and adjusted velocity   =   51374517.826145   PTV2 Eqs (12)/(13)

MAGNETIC POTENTIAL ADJUSTMENT (ALL ENERGIES IN MHz)
(11) Average h-f energy   =   51375882.372400   Horbatsch and Hessels (2016)
(12) Difference between PTV f-s with corrections
    and average h-f energy   =   1364.546255
(13) Exit potential   =   1364.546002   PTV2 Eq (22) & Tables 2/3
(14) H-f mid-point error magnitude   =   0.000254
(15) H-f mid-point PTV   =   51375882.372146

HYPERFINE SHIFT (ALL ENERGIES IN MHz)
(16) Higher h-f energy from experiment   =   51375882.443700   Horbatsch and Hessels (2016)
(17) Higher h-f energy calculated from PTV =   51375882.443472   PTV2 Eqs (11)
(18) Deviation of PTV from experiment is one part in   225669400955.246208
(19) H-f difference taken from experiment =   0.071300
(20) H-f difference calculated from PTV   =   0.071326   PTV2 Eq (37)
(21) H-f error magnitude   =   0.000026

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    mean absolute h-f midpoint =   0.006720
    h-f shift mean absolute error =   0.008219