

' A PROGRAM TO CALCULATE THE HYDROGEN HYPERFINE ENERGY LEVELS USING THE NEW PTV THEORY

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' INSTRUCTIONS

' mm controls the sign of gamma

' nn runs through the states for nn = 1 to 30

' The A and B values come from a separate optimization program

' PTV1: A Photonic toroidal vortex model of the hydrogen atom fine structure

' PTV2: Geometrical interpretation of the hydrogen atom hyperfine structure

,

' CODATA (June 2019)

alpha1 = 0.0072973525693 ' NIST: fine structure constant

alpha2 = alpha1*alpha1 ' square of alpha1

Rydfreq = 3289841960.2508 ' (MHz) NIST: Rydberg frequency, (1/2)*mo*alpha2*cflight^2*10^(-6)/h

Me = 9.1093837015*10^(-31) ' (kg) NIST: electron rest mass

cflight = 299792458 ' (m/s), speed of light, CODATA

M = 0.000544617021488 ' NIST: e- to p+ rest mass ratio

pi = 3.141592653589793

h = 6.62607015*10^(-34) ' (J/Hz) Planck constant, CODATA

' OUTPUT TEMPLATES

template1\$ = "#####.#####"

template2\$ = "#####"

template3\$ = "#####"

print ""

print " Please wait while writing to PTV.txt"

xlogcoord = 0

ylogcoord = 0

xlogcoordstore = 0

ylogcoordstore = 0

xxlogcoordstore = 0

xylogcoordstore = 0

hfmidpointstore = 0

hfmidcount = 0

hfshifterror = 0

counter = 0

open "PTV_TEST.txt" for output as #PTV

print #PTV, ""

print #PTV, TIMES\$(); " ";DATES\$()

print #PTV, ""

print #PTV, "**** A program to calculate hydrogen atom hyperfine levels using the new photonic toroidal vortex (PTV) model ****"

print #PTV, " written in Liberty BASIC by Dr Barry R. Clarke, completed 28 May 2024"

print #PTV, ""

print #PTV, " All equations and tables refer to the following papers."

print #PTV, ""

print #PTV, " PTV1: Clarke, Barry R., A photonic toroidal vortex model of the hydrogen atom fine structure, Quantum Studies:"

print #PTV, " Mathematics and Foundations, 12, article 18 (2025)"

print #PTV, " PTV2: Clarke, Barry R., The hydrogen atom hyperfine structure from a photonic toroidal vortex model,"

print #PTV, " submitted to Foundations of Physics (2025)"

print #PTV, ""

print #PTV, " EXPERIMENTAL DATA"

print #PTV, " Horbatsch, M. and E. A. Hessels, Tabulation of the bound-state energies of atomic hydrogen, Physical Review A 93, 022513 (2016)"

print #PTV, ""

print #PTV,

"

print #PTV, ""

' NIST DATA: Hyperfine energy levels (MHz): Horbatsch and Hessels, Phys. Rev. A, 93 (2016, Tables 3 & 4)

' numbered 1-6 for nS1/2, 7-12 for nP1/2, , 13-18 nP3/2, 19-24 for nD3/2, 25-30 for nD5/2

for mm = 1 to 2

for nn = 1 to 30

select case

case nn = 1

print #PTV, "1S1/2"

NIST0 = 3288087922.4160 '1S(1/2) ionization energy (F=0)

NIST1 = 3288086502.0102 '1S(1/2) ionization energy (F=1)

n = 1

L = 0

Sp = 1/2

case nn = 2

print #PTV, "2S1/2"

NIST0 = 822025577.0922 '2S(1/2) (F=0)

NIST1 = 822025399.5354 '2S(1/2) (F=1)

n = 2

L = 0

Sp = 1/2

case nn = 3

print #PTV, "3S1/2"

NIST0 = 365343617.9043 '3S(1/2) (F=0)

NIST1 = 365343565.2949 '3S(1/2) (F=1)

n = 3

L = 0

Sp = 1/2

case nn = 4

print #PTV, "4S1/2"

NIST0 = 205505309.9525 '4S(1/2) (F=0)

NIST1 = 205505287.7579 '4S(1/2) (F=1)

n = 4

L = 0

Sp = 1/2

case nn = 5

print #PTV, "5S1/2"

NIST0 = 131523180.9882 '5S(1/2) (F=0)

NIST1 = 131523169.6246 '5S(1/2) (F=1)

n = 5

L = 0

Sp = 1/2

case nn = 6

print #PTV, "6S1/2"

NIST0 = 91335431.6017 '6S(1/2) (F=0)

NIST1 = 91335425.0256 '6S(1/2) (F=1)

n = 6

L = 0

Sp = 1/2

' *****

case nn = 7

print #PTV, "2P1/2"

NIST0 = 822026546.1359 '2P(1/2) (F=0)

NIST1 = 822026486.9664 '2P(1/2) (F=1)

n = 2

L = 1

Sp = -1/2

case nn = 8

print #PTV, "3P1/2"

NIST0 = 365343906.4710 '3P(1/2) (F=0)

NIST1 = 365343888.9392 '3P(1/2) (F=1)

n = 3

L = 1
Sp = -1/2

case nn = 9
print #PTV, "4P1/2"
NIST0 = 205505431.9299 '4P(1/2) (F=0)
NIST1 = 205505424.5337 '4P(1/2) (F=1)
n = 4
L = 1
Sp = -1/2

case nn = 10
print #PTV, "5P1/2"
NIST0 = 131523243.5005 '5P(1/2) (F=0)
NIST1 = 131523239.7136 '5P(1/2) (F=1)
n = 5
L = 1
Sp = -1/2

case nn = 11
print #PTV, "6P1/2"
NIST0 = 91335467.7972 '6P(1/2) (F=0)
NIST1 = 91335465.6058 '6P(1/2) (F=1)
n = 6
L = 1
Sp = -1/2

case nn = 12
print #PTV, "7P1/2"
NIST0 = 67103543.4423 '7P1/2
NIST1 = 67103542.0628
n = 7
L = 1
Sp = -1/2

' *****

case nn = 13
print #PTV, "2P3/2"
NIST0 = 822015547.4967 '2P(3/2) (F=1)
NIST1 = 822015523.8451 '2P(3/2) (F=2)
n = 2
L = 1
Sp = 1/2

case nn = 14
print #PTV, "3P3/2"
NIST0 = 365340647.6119 '3P(3/2) (F=1)
NIST1 = 365340640.6040 '3P(3/2) (F=2)
n = 3
L = 1
Sp = 1/2

case nn = 15
print #PTV, "4P3/2"
NIST0 = 205504057.1004 '4P(3/2) (F=1)
NIST1 = 205504054.1439 '4P(3/2) (F=2)
n = 4
L = 1
Sp = 1/2

case nn = 16
print #PTV, "5P3/2"
NIST0 = 131522539.5886 '5P(3/2) (F=1)
NIST1 = 131522538.0749 '5P(3/2) (F=2)
n = 5
L = 1
Sp = 1/2

case nn = 17
print #PTV, "6P3/2"
NIST0 = 91335060.4413 '6P(3/2) (F=1)
NIST1 = 91335059.5653 '6P(3/2) (F=2)
n = 6
L = 1
Sp = 1/2

case nn = 18 ' Table IV
print #PTV, "7P3/2"
NIST0 = 67103286.9157
NIST1 = 67103286.3640
n = 7
L = 2
Sp = -1/2

' *****

case nn = 19
print #PTV, "3D3/2"
NIST0 = 365340651.1931 '3D(3/2) (F=1)
NIST1 = 365340646.9865 '3D(3/2) (F=2)
n = 3
L = 2
Sp = -1/2

case nn = 20
print #PTV, "4D3/2"
NIST0 = 205504058.6495 '4D(3/2) (F=1)
NIST1 = 205504056.8748 '4D(3/2) (F=2)
n = 4
L = 2
Sp = -1/2

case nn = 21
print #PTV, "5D3/2"
NIST0 = 131522540.3924 '5D(3/2) (F=1)
NIST1 = 131522539.4837 '5D(3/2) (F=2)
n = 5
L = 2
Sp = -1/2

case nn = 22
print #PTV, "6D3/2"
NIST0 = 91335060.9101 '6D(3/2) (F=1)
NIST1 = 91335060.3843 '6D(3/2) (F=2)
n = 6
L = 2
Sp = -1/2

case nn = 23 ' Table V
print #PTV, "7D3/2"
NIST0 = 67103287.2124
NIST1 = 67103286.8813
n = 7
L = 2
Sp = -1/2

case nn = 24
print #PTV, "8D3/2"
NIST0 = 51375939.6283
NIST1 = 51375939.4065
n = 8
L = 2
Sp = -1/2

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' *****
case nn = 25
print #PTV, "3D5/2"
NIST0 = 365339566.8037 '3D(5/2) (F=2)
NIST1 = 365339564.1003 '3D(5/2) (F=3)
n = 3
L = 2
Sp = 1/2
case nn = 26
print #PTV, "4D5/2"
NIST0 = 205503601.1722 '4D(5/2) (F=2)
NIST1 = 205503600.0315 '4D(5/2) (F=3)
n = 4
L = 2
Sp = 1/2
case nn = 27
print #PTV, "5D5/2"
NIST0 = 131522306.1639 '5D(5/2) (F=2)
NIST1 = 131522305.5800 '5D(5/2) (F=3)
n = 5
L = 2
Sp = 1/2
case nn = 28
print #PTV, "6D5/2"
NIST0 = 91334925.3612 '6D(5/2) (F=2)
NIST1 = 91334925.0233 '6D(5/2) (F=3)
n = 6
L = 2
Sp = 1/2
case nn = 29 ' 7D5/2
print #PTV, "7D5/2"
NIST0 = 67103201.8522
NIST1 = 67103201.6394
n = 7
L = 2
Sp = 1/2
case nn = 30 ' 8D5/2
print #PTV, "8D5/2"
NIST0 = 51375882.4437
NIST1 = 51375882.3011
n = 8
L = 2
Sp = 1/2

```

end select

' hyperfine k values, PTV2 Table 7

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select case
case nn>=1 and nn<=6 ' nS1/2
mult = 1
case nn>=7 and nn<=12 ' nP1/2
mult = 1/3
case nn>=13 and nn<=18 ' nP3/2
mult = 2/15
case nn>=19 and nn<=24 ' nD3/2
mult = 2/25
case nn>=25 and nn<=30 ' nD5/2
mult = 25/486
end select

```

' Sommerfeld fine structure quantum numbers

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na = L + Sp + 0.5 ' azimuthal quantum number
nr = n - na ' radial quantum number
Y = nr + sqrt(na*na - alpha2) ' PTV1 Eq. (39)
Y2 = Y*Y

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print #PTV, "QUANTUM NUMBERS"
print #PTV, "(1) Quantum Mechanics: n = ";n;" L = ";L;" Sp = ";Sp;" Sommerfeld: n(phi) = ";na;" n(r) = ";nr
print #PTV, ""
gosub [Sommerfeldfs]
gosub [coordinates] ' calculates r2f2 and gives coordinates to evel3
print #PTV, "BOUND STATE DISTANCES (as a multiplier of a fixed radius for all states)"
print #PTV, "(2) Horizontal distance between p-e Sp-2 centers = ";using (template3$, Xstore); " PTV1 x-bar Eqs (84)/(86)"
r2f2 = Xstore^2 + (1 - M)^2
print #PTV, "(3) Shortest distance between p-e Sp-2 centers = ";using (template3$,sqr(r2f2)); " PTV1 d2-bar Eq (86)"

krel = sqr(1 - alpha2/(2*(Y^2 + alpha2)))

gosub [emultx]

' WE DON'T TAKE THIS AS RELATIVISTIC BECAUSE IT'S PURE Sp-2
evel3 = (1 + emult*M/n)^2 ' nS1/2

print #PTV, ""
gosub [MVRfs2]
gosub [Hfmodel]
gosub [coordinates]
gosub [redshift]
gosub [printout1]

next nn
next mm

print #PTV, " mean absolute h-f midpoint = "; using (template1$, hfmidpointstore/60)
print #PTV, " h-f shift mean absolute error = "; using (template1$, hfshifterror/hfmidcount)
print #PTV, " *** Free license, Barry R. Clarke (aleteller@barryispuzzled.com), 28 May 2024 ***"
close #PTV
print " "
print " Finished"
end

' *****
' Calculates Sommerfeld's fine structure energies with and without reduced mass in PTV1 Table 1
[Sommerfeldfs]
redmass1 = 1/(1 + M) ' reduced mass based on rest masses
Somm01 = na^2 - alpha2 ' na is Sommerfeld's azimuthal quantum number
Somm02 = sqr(Somm01)
Somm03 = nr ' Sommerfeld's radial quantum number
Somm04 = alpha1/(Somm02 + Somm03)
Somm05 = 1 + Somm04^2
Somm06 = 1/(sqr(Somm05))
Somm07 = 1 - Somm06
SommEnergy = Somm07*redmass1*Rydfreq*2/alpha2
return

' PTV fine structure energy

[MVRfs2]

evel3a = nr + sqr(na*na - alpha2)
evel3b = evel3a*evel3a
evel3c = alpha2/(evel3b)
evel3d = evel3c/(2*(1 + evel3c))
evel3e = sqr(1 - evel3d)
evel3f = evel3b + alpha2
k1 = M/(evel3e)
redmass2 = 1/(1 + k1) ' reduced mass with relativistic electron mass

evel3aa = nr + sqr(na*na - alpha2*evel3)
evel3bb = evel3aa*evel3aa

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evel3cc = alpha2*evel3/(evel3bb)
evel3dd = evel3cc/(2*(1 + evel3cc))
evel3ee = sqrt(1 - evel3dd)
evel3ff = evel3bb + alpha2*evel3
k2 = M/evel3ee
redmass3 = 1/(1 + k2)

VortexEnergy0 = Rydfreq/(evel3e*evel3f)          ' PTV fine structure with no reduced mass
VortexEnergy1 = redmass3*Rydfreq/(evel3ee*evel3ff)  ' PTV fine structure with reduced mass
VortexEnergy2 = redmass3*evel3*Rydfreq/(evel3ee*evel3ff) ' PTV fine structure with reduced mass & D function PTV2 Eqs (12)/(13)

return

' Proton-electron separation for ring energy equalling intruding proton field energy in C of M frame
' Brute force search for x displacement relative to proton Sp-2 center PTV1 Eq. (85)
[coordinates]

Xinc = 0          ' initialize increment for X search
Xstore = 0       ' initialize memorized value of X
for acc = -1 to 14 ' variable controlling decimal place searched
for msearch = 0 to 150 ' raised upper limit for high x, only 0-10 reqd for decimal places

' normalised values, divided by r3o
Xinc = msearch/10^(acc) ' increment added to previous trial X
X = Xstore + Xinc      ' stores trial value of X coordinate, as fraction of ground state radius
XX = X                ' first equation (7.54) (The Vortex Atom: A New Paradigm)
R1 = M                ' second equation (7.54)
R2 = alpha1/sqrt(1 - alpha2)

' left side of PTV1 Eq. (85)
FX1 = 1
FX2 = n*sqrt(1 + alpha2/Y^2)*(1 - alpha2)
FX3 = FX1/FX2
Fint1 = 1 - R1
Fint2 = 1 + (XX*XX + R2*R2)/(Fint1*Fint1)
Fint3 = 1/(Fint1*sqrt(Fint2))
Fint4 = R2*R2*(XX*XX + Fint1*Fint1)/(Fint1^4*Fint2^2)
Fint5 = Fint4*Fint4
Fint = Fint3*(1 + (3/4)*Fint4 + (105/64)*Fint5)
FX = FX3*Fint

' right side of PTV1 Eq. (85)
Felectron = (0.5/(Y^2 + alpha2))*(1/sqrt(1 - alpha2/(2*(Y^2 + alpha2))))

' minimize Fsearch
Fsearch = FX - Felectron

if Fsearch < 0 then ' attach marker value to sign of Fsearch
mark = 0
else
mark = 1
end if

if msearch > 0 then ' only compare Fsearch sign with previous one only after first pass
if mark <> storemark then
Xstore = Xstore + (msearch - 1)/10^acc ' return to previous digit in search
msearch = 10
end if
end if

storemark = mark ' store present Fsearch sign

next msearch
next acc

return

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' Hyperfine separation from mid-point PTV2 Eq. (37)
[Hfmodel]
' NO CORRECTIONS FOR D FUNCTION
Hfsummary1 = 2*n^2*pi*Rydfreq*alpha1^3
Hfsummary2 = sqrt(r2f2)^3*(1 - alpha2)^(3/2)*Y*sqr(Y^2 + alpha2)*sqr(2)
Hfshift = mult*Hfsummary1/Hfsummary2

return

' emult is gamma in Eq. (13)
[emultx]
select case
case (nn >= 1 and nn <= 6)
if mm = 1 then
emult = 1.006942700
end if
if mm = 2 then
emult = -0.549012
end if

case (nn >= 7 and nn <= 12)

if mm = 1 then
emult = 0.55773890261
end if
if mm = 2 then
emult = -0.20481644002
end if

case (nn >= 13 and nn <= 18)
if mm = 1 then
emult = 0.48878536995
end if
if mm = 2 then
emult = -0.23361014241
end if

case (nn >= 19 and nn <= 24)
if mm = 1 then
emult = 0.3957755
end if
if mm = 2 then
emult = -0.1836735
end if

case (nn >= 25 and nn <= 30)
if mm = 1 then
emult = 0.37459705
end if
if mm = 2 then
emult = -0.1950811
end if
end select
return

[printout1]

print #PTV, " emult = ";using(template2$, emult)
print #PTV, ""
print #PTV, "FINE STRUCTURE (ALL ENERGIES IN MHz)"
print #PTV, "(4) Sommerfeld f-s energy no adjustments = ";using (template1$, SommEnergy/redmass1)
print #PTV, "(5) New PTV f-s energy no adjustments = ";using (template1$, VortexEnergy0);" PTV1 Eq (54)"
print #PTV, "(6) Sommerfeld f-s energy reduced mass = ";using (template1$, SommEnergy)

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print #PTV, "(7) Relativistic reduced mass multiplier = ";using (template3$,redmass3); " PTV2 (1 + Mr)(-1) Eq (13)"
print #PTV, "(8) New PTV f-s energy relativ. reduced mass = ";using (template1$, VortexEnergy1);" PTV2 (D = 1) Eq (13)"
print #PTV, "(9) Velocity D multiplier = ";evel3; " PTV2 Eq (13)"
print #PTV, "(10) New PTV f-s energy with relativistic reduced mass "
print #PTV, " and adjusted velocity = ";using (template1$, VortexEnergy2); " PTV2 Eqs (12)/(13)"
print #PTV, ""
print #PTV, "EXIT POTENTIAL RED-SHIFT (ALL ENERGIES IN MHz)"
print #PTV, "(11) Average h-f energy = "; using (template1$, (NIST0 + NIST1)/2);" Horbatsch and Hessels (2016)"
print #PTV, "(12) Difference between PTV f-s with corrections"
print #PTV, " and average h-f energy = "; using (template1$, (abs(VortexEnergy2 - (NIST0 + NIST1)/2)));
print #PTV, "(13) Exit potential = "; using (template1$, exitsh1);" PTV2 Eq (22) & Tables 2/3"

if emult > 0 then ' F=1 (low freq)

partscalc0 = NIST1/( NIST1 - (VortexEnergy2 - Hfshift - exitsh1))

aa01 = abs(VortexEnergy2 - (NIST0 + NIST1)/2 - exitsh1)
print #PTV, "(14) H-f mid-point error magnitude = "; using (template1$, aa01)
print #PTV, "(15) H-f mid-point PTV = "; using (template1$, (abs(VortexEnergy2 - exitsh1)))

print #PTV, ""
print #PTV, "HYPERFINE SHIFT (ALL ENERGIES IN MHz)"
print #PTV, "(16) Lower h-f energy from experiment = "; using (template1$, NIST1);" Horbatsch and Hessels (2016)"
print #PTV, "(17) Lower h-f energy calculated from PTV = "; using (template1$, VortexEnergy2 - exitsh1 - Hfshift)
print #PTV, "(18) Deviation of PTV from experiment is one part in ";using (template1$, abs(partscalc0))

print #PTV, "(19) H-f difference taken from experiment = ";using (template1$, (NIST0 - NIST1)/2)
print #PTV, "(20) H-f difference calculated from PTV = ";using (template1$, Hfshift)
print #PTV, "(21) H-f error magnitude = ";using (template1$, abs(Hfshift - (NIST0 - NIST1)/2))

hfmidpointstore = hfmidpointstore + aa01
counter = counter + 1
end if
' *****
if emult < 0 then ' F=0 (high freq)

partscalc1 = NIST0/( NIST0 - (VortexEnergy2 + Hfshift + exitsh1))

bb01 = abs(VortexEnergy2 - (NIST0 + NIST1)/2 + exitsh1)
print #PTV, "(14) H-f mid-point error magnitude = "; using (template1$, bb01)
print #PTV, "(15) H-f mid-point PTV = "; using (template1$, (abs(VortexEnergy2 + exitsh1)))
print #PTV, ""
print #PTV, "HYPERFINE SHIFT (ALL ENERGIES IN MHz)"
print #PTV, "(16) Higher h-f energy from experiment = "; using (template1$, NIST0);" Horbatsch and Hessels (2016)"
print #PTV, "(17) Higher h-f energy calculated from PTV = "; using (template1$, VortexEnergy2 + exitsh1 + Hfshift)
print #PTV, "(18) Deviation of PTV from experiment is one part in ";using (template1$, abs(partscalc1))

print #PTV, "(19) H-f difference taken from experiment = ";using (template1$, (NIST0 - NIST1)/2)
print #PTV, "(20) H-f difference calculated from PTV = ";using (template1$, Hfshift)
print #PTV, "(21) H-f error magnitude = ";using (template1$, abs(Hfshift - (NIST0 - NIST1)/2))

hfmidpointstore = hfmidpointstore + bb01
counter = counter + 1

end if

if mm = 1 then
hfshifterror = hfshifterror + abs(Hfshift - (NIST0 - NIST1)/2)
hfmidcount = hfmidcount + 1
end if

print #PTV, ""
print #PTV, ""
"
print #PTV, ""

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return
[redshift]
select case
case (nn >= 1 and nn <= 6)
if emult > 0 then
A = 28923705.16563
exitsh1 = A*(sqr(r2f2))(-3.0000421859595)
end if
if emult < 0 then
A = 15666540.39306
exitsh1 = A*(sqr(r2f2))(-2.999976494243)
end if

case (nn >= 7 and nn <= 12)

if emult > 0 then
A = 15981616.6169
exitsh1 = A*(sqr(r2f2))(-3.00000946384)
end if
if emult < 0 then
A = 5870876.7450
exitsh1 = A*(sqr(r2f2))(-3.000001596923)
end if

case (nn >= 13 and nn <= 18)
if emult > 0 then
A = 14008000.6385
exitsh1 = A*(sqr(r2f2))(-3.00000976361)
end if
if emult < 0 then
A = 6693635.451
exitsh1 = A*(sqr(r2f2))(-3.000002303498)
end if

case (nn >= 19 and nn <= 24)
if emult > 0 then
A = 11341280.0376
exitsh1 = A*(sqr(r2f2))(-3.000004673972)
end if
if emult < 0 then
A = 5263782.149
exitsh1 = A*(sqr(r2f2))(-3.000002402484)
end if

case (nn >= 25 and nn <= 30)
if emult > 0 then
'A = 11957572.978
A = 10734985.1348
exitsh1 = A*(sqr(r2f2))(-3.000004778242)
end if
if emult < 0 then
'A = 7718531.9656
A = 5590077.3226
exitsh1 = A*(sqr(r2f2))(-3.0000023717)
end if
end select

return
' *** END OF PROGRAM ***

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